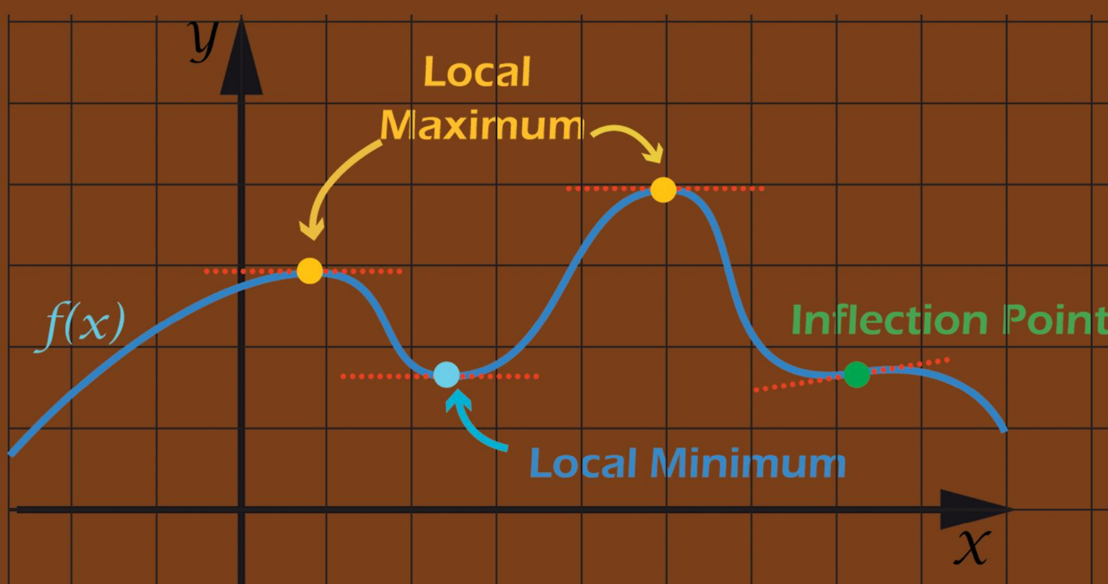
Combined Mathematics

Grade 12

Teachers' Guide

(To be Implemented from 2017)



Department of Mathematics
Faculty of Science & Technology
National Institute of Education
Sri Lanka

Printing & Distribution: Educational Publication Department

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Combined Mathematics
Grade 12 - Teachers' Guide

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Content

Message from the Director General

With the primary objective of realizing the National Educational Goals recommended by the National Education Commission, the then prevalent content based curriculum was modernized, and the first phase of the new competency based curriculum was introduced to the eight year curriculum cycle of the primary and secondary education in Sri Lanka in the year 2007

The second phase of the curriculum cycle thus initiated was introduced to the education system in the year 2015 as a result of a curriculum rationalization process based on research findings and various proposals made by stake holders.

Within this rationalization process the concepts of vertical and horizontal integration have been employed in order to build up competencies of students, from foundation level to higher levels, and to avoid repetition of subject content in various subjects respectively and furthermore, to develop a curriculum that is implementable and student friendly.

The new Teachers' Guides have been introduced with the aim of providing the teachers with necessary guidance for planning lessons, engaging students effectively in the learning teaching process, and to make Teachers' Guides will help teachers to be more effective within the classroom. Further, the present Teachers' Guides have given the necessary freedom for the teachers to select quality inputs and activities in order to improve student competencies. Since the Teachers' Guides do not place greater emphasis on the subject content prescribed for the relevant grades, it is very much necessary to use these guides along with the text books compiled by the Educational Publications Department if, Guides are to be made more effective.

The primary objective of this rationalized new curriculum, the new Teachers' Guides, and the new prescribed texts is to transform the student population into a human resource replete with the skills and competencies required for the world of work, through embarking upon a pattern of education which is more student centered and activity based.

I wish to make use of this opportunity to thank and express my appreciation to the members of the Council and the Academic Affairs Board of the NIE the resource persons who contributed to the compiling of these Teachers' Guides and other parties for their dedication in this matter.

Dr. (Mrs.) Jayanthi Gunasekara
Director General
National Institute of Education

Message from the Deputy Director General

Education from the past has been constantly changing and forging forward. In recent years, these changes have become quite rapid. The Past two decades have witnessed a high surge in teaching methodologies as well as in the use of technological tools and in the field of knowledge creation.

Accordingly, the National Institute of Education is in the process of taking appropriate and timely steps with regard to the education reforms of 2015.

It is with immense pleasure that this Teachers' Guide where the new curriculum has been planned based on a thorough study of the changes that have taken place in the global context adopted in terms of local needs based on a student-centered learning-teaching approach, is presented to you teachers who serve as the pilots of the schools system.

An instructional manual of this nature is provided to you with the confidence that, you will be able to make a greater contribution using this.

There is no doubt whatsoever that this Teachers' Guide will provide substantial support in the classroom teaching-learning process at the same time. Furthermore the teacher will have a better control of the classroom with a constructive approach in selecting modern resource materials and following the guide lines given in this book.

I trust that through the careful study of this Teachers Guide provided to you, you will act with commitment in the generation of a greatly creative set of students capable of helping Sri Lanka move socially as well as economically forward.

This Teachers' Guide is the outcome of the expertise and unflagging commitment of a team of subject teachers and academics in the field Education.

While expressing my sincere appreciation for this task performed for the development of the education system, my heartfelt thanks go to all of you who contributed your knowledge and skills in making this document such a landmark in the field.

M.F.S.P. Jayawardhana
Deputy Director General
Faculty of Science and Technology

Guidlines to use the Teachers' Guide

In the G.C.E (A/L) classes new education reforms introduced from the year 2017 in accordance with the new education reforms implemented in the interim classes in the year 2015. According to the reforms, Teachers' Guide for combine mathematics for grade 12 has been prepared.

The grade 12 Teacher's Guide has been organized under the titles competencies and competency levels, content, learning outcomes and number of periods. The proposed lesson sequence is given for the leaning teaching process. Further it is expected that this teachers' Guide will help to the teachers to prepare their lessons and lessons plans for the purpose of class room learning teaching process. Also it is expected that this Guide will help the teachers to take the responsibility to explains the subject matters more confidently. This teachers' Guide is divided into three parts each for a term.

In preparing lesson sequence, attention given to the sequential order of concepts, students ability of leaning and teachers ability of teaching. Therefore sequential order of subject matters in the syllabus and in the teachers Guide may differ. It is adviced to the teachers to follow the sequence as in the teachers' Guide.

To attain the learning outcomes mentioned in the teachers' Guide, teachers should consider the subject matters with extra attention. Further it is expected to refer extra curricular materials and reference materials to improve their quality of teaching. Teachers should be able to understand the students, those who are entering grade 12 classes to learn combined mathematics as a subject. Since G.C.E (O/L) is designed for the general education, students joining in the grade 12 mathematics stream will face some difficulties to learn mathematics. To over come this short coming an additional topics on basic Algebra and Geometry are added as pre request to learn. For this purpose teachers can use their self prepared materials or "A beginners course in mathematics" book prepared by NIE.

Total number of periods to teach this combined mathematics syllabus is 600. Teachers can be flexible to change the number of periods according to their necessity. Teachers can use school based assessment to assess the students.

The teacher has the freedom to make necessary amendments to the specimen lesson plan given in the new teacher's manual which includes many new features, depending on the classroom and the abilities of the students.

We would be grateful if you would send any amendments you make or any new lessons you prepare to the Director, Department of Mathematics, National Institute of Education. The mathematics department is prepared to incorporate any new suggestions that would advance mathematics education in the upper secondary school system.

S. Rajendram
Project Leader
Grade 12-13 Mathematics

Approval:	**Academic Affairs Board** National Institute of Education
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	Mr. M. F. S. P. Jayawardana Deputy Director General Facult of Science and Technology National Institute of Education
Supervision :	**Mr. K. R. Pathmasiri** Director, Department of Mathematics National Institute of Education
Subject Coordination:	**Mr. S. Rajendram** Project Leader (Grade 12-13 Mathematics) Department of Mathematics National Institute of Education
	Miss. K. K.Vajeema S. Kankanamge Assistant Lecturer, Department of Mathematics National Institute of Education

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Ms. D. A. D. Withanage	Teacher Srimavo Bandaranayake Vidyalaya, Colombo 07
Mr. W. Kapila Peris	Engineer National Engineering Institute for Research and Development

Other Resource Persons of Department of Mathematics:

Mr. G.P.H. Jagath Kumara	Senior Lecturer National Institute of Education
Mr. G. L. Karunarathna	Senior Educatanist National Institute of Education
Ms. M. Nilmini P. Peiris	Senior Lecturer National Institute of Education
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Miss. K.K.Vajeema S. Kankanamge	Assistant Lecturer National Institute of Education

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First Term

Combined Mathematics I

Competency 1 : Analyses the system of real numbers

Competency level 1.1 : Classifies the set of real numbers

Number of periods : 01

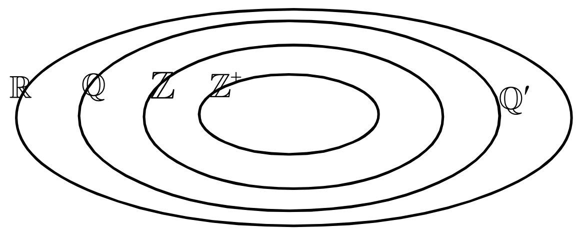
Learning outcomes :
1. Explains the evolution of the number system.
2. Introduces notations for sets of numbers.
3. Represents a real number geometrically.

Guidelines to learning - teaching process :

1. Explain briefly the evolution from the inception of the use of numbers upto the real number system.
2. Recall the knowledge of pupils about the sets of natural numbers, integers, rational numbers, irrational numbers and real numbers.
   - The set of Intergers $\mathbb{Z} = \{\ldots, -5, -4, -3, -2, -1, 0, 1, 2, \ldots\}$
   - The set of positive integers (Natural numbers)

     $Z^+ = \{1, 2, 3, \ldots\}$
   - The set of Real numbers $\mathbb{R}$
   - The set of Rational numbers

     $\mathbb{Q} = \left\{ x : x = \frac{p}{q},\ q \neq 0,\ p, q \in \mathbb{Z} \right\}$
   - The set of Irrational numbers Q^1
   - Explains that all of the above sets are sub sets of $\mathbb{R}$
   - Direct the students to denote them in a Venn Diagram.



3. Remind the representation of a real number on a number line.

- Guide students to mark the following numbers in the number line
  - Rational numbers
  - Irational numbers

Competency level 1.2 : Uses surds or decimals to describe real numbers

Number of periods : 01

Learning outcomes :
1. Classifies decimal numbers
2. Rationalises the denominator of expressions with surds.

Guidelines to learning - teaching process :

1. • Decimal Numbers
   - Finite decimals
   - Infinite decimals
     - Recurring decimals
     - Non-recurring decimals

   • Real Numbers
   - Rational numbers
   - Irrational numbers

2. • Introduce surds as solutions of an equation.

   • Mainpulates algebraic operation on surds.
     - Addition
     - Subtraction
     - Multiplication
     - Division

   • Guide students to simplify problems involving surds.

Competency 2 : Analyses single variable functions

Competency level 2.1 : 2.1 Review of functions

Number of periods : 02

Learning outcomes :

1. Explains the intuitive idea of a function.
2. Recognizes constants, variables
3. Explains relationship between two variables
4. Explains domain and codomain
5. Explains one - one functions
6. Explains onto functions
7. Explains inverse functions

Guidelines to learning - teaching process :

1. Introduce functions through illustrations
2. Introduce constants and variable
   - Explain with examples one-one, one-many, many - one, and many-many relations between two sets.
3. Function f from a set X in to set Y is a rule which corresponds each element x in X to an unique element y in Y.
4. Introduce independent variable, dependent variable, image, Domain (D), Codomain (C) and Range (R) of a function and, functional notation $f : \mathrm{X} \to \mathrm{Y}$

   $$y = f(x).$$
5. Explains one - one functions through illustrations
   - Horizontal line test for one - one functions
6. Explains onto functions through examples.
7. Explains inverse function through examples.
   - Guide students to find inverse functions (simple examples)

Competency level 2.2 : Reviews types of functions

Number of periods : 02

Learning outcomes :
1. Recognizes special functions
2. Sketches graph of functions
3. Finds composite functions

Guidelines to learning-teaching process :

1. Introduce constant function, linear function, modulus function, piecewise function.
   - Constant function : $f(x) = k$, where k is a constant

     $f(x)$ is said to be unit function when $k = 1$

     Illustrate the above through examples
   - Modulus (absolute value) function.

     $$f(x) = |x| = \begin{cases} x \ ; \ x \geq 0 \\ -x \ ; \ x < 0 \end{cases}$$

     Sketches graphs of modulus functions
   - Piecewise function: Functions where the rule of the function changes for various intervals of the domain.

     Eg. $f(x) = \begin{cases} x+1, \ x > 0 \\ 5, \ x = 0 \\ -x, \ x < 0 \end{cases}$
   - Sketches graphs and explains.
2. Graph of a function :
   Stress the vertical line test . A line parallel to y axis cuts the graph of a function only at one point.
3. Composite functions :
   - Let f and g be functions of x. Then functions h, t such that $h(x) = f[g(x)]$ and $t(x) = g[f(x)]$ are said to be composite functions.
   - Explains composite functions using examples.

Competency 8 : Uses the relations involving angular measure

Competency level 8.1 : State the relationship between radian and degree.

Number of periods : 01

Learning outcomes :
1. Introduces degree and radian as units of measurement of angles
2. Convert degree into radians and vice-versa.

Guidelines to learning teaching process :

1.
   - States that the units used to measure angles are degree and radian.
   - Finds relationship between radians and degrees.
   - Define degree and radian.

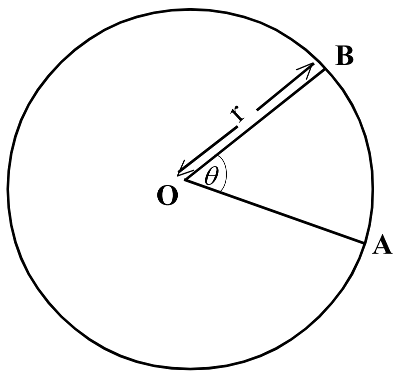
2.
   - Convert degree into radian and vice-versa.

Competency level 8.2 : Solves problems involving arc length and area of a circular sector

Number of periods : 01

Learning outcomes :
1. Finds the length of an arc and area of a circular sector.

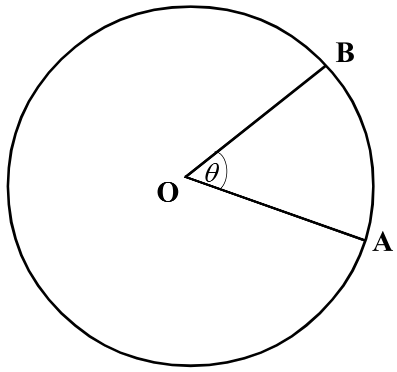
Guidelines to learning - teaching process :

1.
   - Introduce that the length S of an arc, subtending an angle θ at the centre of a circle with radius r is given by $S = r\theta$, where θ measured in radians.
   - 

length of the arc $AB = r\theta$

$$S = r\theta$$

- Introduce that the area A of a sector, subtending an angle θ at the centre of a circle with radius r is given by $A = \frac{1}{2} r^2 \theta$, where θ measured in radians.



- Area of the sector OAB $= \frac{1}{2} r^2 \theta$

Competency 17 : **Uses the rectangular system of Cartesian axes and simple geometrical results.**

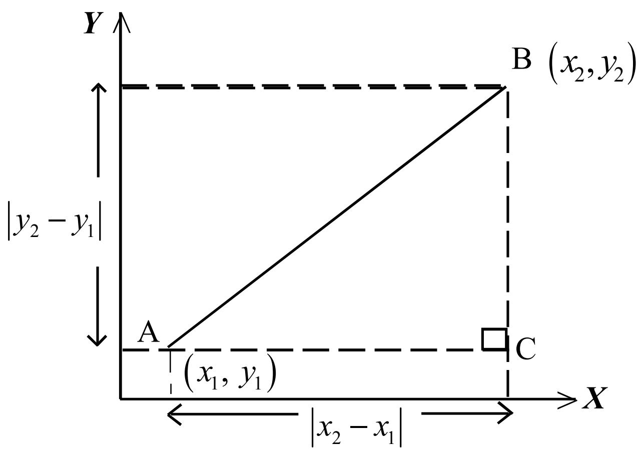
Competency level 17.1 : Finds the distance between two points on the Cartesian plane

Number of periods : 01

Learning outcomes :
1. Explains the Cartesian coordinate system
2. Defines the abscissa and the ordinate.
3. Introduces the four quadrants in the cartesian coordinate plane.
4. Finds the length of a line segment joining two points.

Guidelines to learning - teaching process :

1. Revise the Cartesian coordinate plane. Explain that X and Y axes are a pair of number lines.
2. Introduce abscissa and the ordinate of the point $P \equiv (x,\ y)$.
3. Introduce the four quadrants in the cartesian coordinate plane.
4. Find that if $A \equiv (x_1,\ y_1)$ and $B \equiv (x_2,\ y_2)$ then



$$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}\ .$$

- Solves problem involving distance between two pints.

Competency level 17.2 : Finds coordinates of the point dividing the straight line segment joining two given points in a given ratio.

Number of periods : 02

Learning outcomes :

1. Finds the coordinates of the point that divides a line segment joining two given points internally in a given ratio.

2. Finds coordinates of the point dividing the straight line segment joining two given points externally in a given ratio.

Guidelines to learning - teaching process :

1. The coordinates of a point P dividing the line segment AB where $A \equiv (x_1, y_1)$ and $B \equiv (x_2, y_2)$ in the ratio $AP : PB = m{:}n$ internally is given by,

$$P \equiv \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

2. The co-ordinate of a point P dividing the line segment AB where $A \equiv (x_1, y_1)$ and $B \equiv (x_2, y_2)$ in the ratio AP : PB = $m{:}n$ externaly is given by,

$$P \equiv \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n} \right) \text{ where } m \neq n$$

- Discuss the cases $m>n$ and $m<n$
- Guide students to finds coordinate of the centroid of a triangle.
- Guide students to solves ploblems involving above results.

Competency 09 : Interprets circular functions

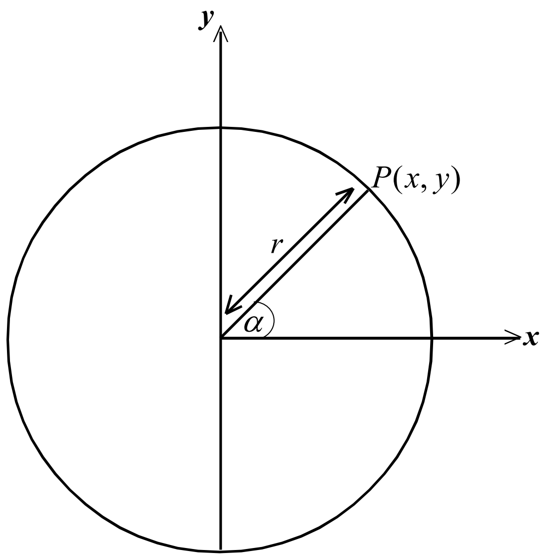
Competency level 9.1 : Decribes basic trigonometric (circular) functions.

Number of periods : 04

Learning outcomes :
1. Explains trigonometric ratios.
2. Defines basic circular (trigonometric) functions.
3. Introduces the domain and the range of circular functions.

Guidelines to learning - teaching process :

1. Define trigonometric ratios using the cartesian coordinate system.



$\sin\alpha = \dfrac{y}{r}$

$\cos\alpha = \dfrac{x}{r}$

$\tan\alpha = \dfrac{y}{x}$; $x \neq 0$

2. Show that trigonometric ratio of a variable angle is a function of that angle. Introduce these ratios as circular functions.

3. Introduce the domain and the range of circular functions.

$y = \sin x$; Domain $= \mathbb{R}$,
Range $= [-1, 1]$

$y = \cos x$; Domain $= \mathbb{R}$,
Range $= [-1, 1]$

$y = \tan x$; Domain $= \mathbb{R}$ - {odd multiples of $\dfrac{\pi}{2}$}
Range $= (-\infty, \infty)$

Competency level 9.2 : Derives values of basic trignometric functions at commonly used angles.

Number of periods : 01

Learning outcomes :
1. Finds the values of functions at given angles.
2. States the sign of basic trigonometric function of θ in each quadrant.

Guidelines to learning - teaching process :

1. Find the values of sin, cos and tan for the following angles.

$$0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$$

2. Show that when θ is in the

    i. First quadrant $\left[0 < \theta < \frac{\pi}{2}\right]$

    $\sin\theta > 0, \cos\theta > 0, \tan\theta > 0$

    Discuss the cases when $\theta = 0$ and $\theta = \frac{\pi}{2}$.

    ii. Second quadrant $\left[\frac{\pi}{2} < \theta < \pi\right]$

    $\sin\theta > 0, \cos\theta < 0, \tan\theta < 0$

    Discuss the cases when $\theta = \frac{\pi}{2}$ and $\theta = \pi$

    iii. Third quadrant $\left[\pi < \theta < \frac{3\pi}{2}\right]$

    $\sin\theta < 0, \cos\theta < 0, \tan\theta > 0$

    Discuss the cases when $\theta = \pi$ and $\theta = \frac{3\pi}{2}$

    iv. Fourth quadrant $\left[\frac{3\pi}{2} < \theta < 2\pi\right]$

    $\sin\theta < 0, \cos\theta > 0, \tan\theta < 0$

    Discuss the cases when $\theta = \frac{3\pi}{2}$ and $\theta = 2\pi$.

    v. Show the above results concisely as follows.

(2) sine (+)	(1) all (+)
(3) tangent (+)	(4) cosine (+)

Competency level 9.3 : Derives the values of basic trignometric functions at angles differing by odd multiples of $\frac{\pi}{2}$ and integer multiples of π.

Number of periods : 03

Learning outcomes :
1. Describes the periodic properties of circular functions
2. Describes the trigonometric relations.
3. Find the values of circular functions at given angles.

Guidelines to learning - teaching process :

1. When an angle is increased by an integer multiple of 2π, the radius vector reaches the initial position after a single or more revolutions. Therefore θ and $2n\pi+\theta$ have the same trigonometric ratios.

2. Obtain the trigonometric ratios of $(-\theta)$, $\frac{\pi}{2}\pm\theta$, $\pi\pm\theta$, $\frac{3\pi}{2}\pm\theta$, $2\pi\pm\theta$ in terms of the trigonometric ratios of θ using geometrical methods.

3. Direct the students to find the values of sin, cos and tan of the angles $\frac{2\pi}{3}$, $\frac{3\pi}{4}$, $\frac{5\pi}{6}$, $\frac{7\pi}{6}$, ...

Competency level 9.4 : Describes the behavior of basic trigonometric functions graphically.

Number of periods : 04

Learning outcomes :
1. Represents the circular functions graphically.
2. Draws graphs of combined circular functions.

Guidelines to learning - teaching process :

1. Introduce the graphs of $\sin x$, $\cos x$ and $\tan x$
2. Direct students to sketch the graphs of

- $y=\sin(x+\alpha)$, $y=\cos(x+\alpha)$, $y=\tan(x+\alpha)$
- $y=\sin kx$, $y=\cos kx$, $y=\tan kx$
- $y=a+b\sin kx$, $y=a+b\cos kx$, $y=a+b\tan kx$
- $y=\sin(kx+b)$, $y=\cos(kx+b)$, $y=\tan(kx+b)$
- $y=a+b\sin(kx+\alpha)$, $y=a+b\cos(kx+\alpha)$, $y=a+b\tan(kx+\alpha)$

Competency 11 : Applies sine rule and cosine rule to solve trigonometric problems.

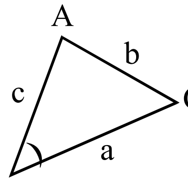
Competency level 11.1 : States Sine rule and Cosine rule.

Number of periods : 01

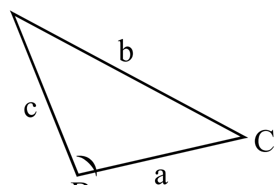
Learning outcomes :
1. Introduces usual notations for a triangle.
2. States sine rule for any triangle.
3. State cosine rulls for any triangle.

Guidelines to learning - teaching process :

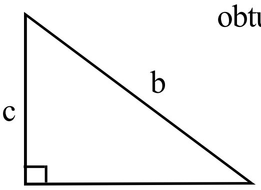
1. State that the angles of a triangle ABC are denoted as A, B and C and the lengths of sides opposite to these angles as a, b and c respectively.



acute angled triangle



obtuse angled triangle



right angled triangle

2. Sine rule
   for any triangle

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

3. Cosine rule
   for any triangle

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

Note : Problems involving these rule are not expected here. but applications in statics are expected.

Competency 4 : Manipulates Polynomial functions.

Competency level 4.1 : Explores polynomials of a single variable.

Number of periods : 01

Learning outcomes :
1. Defines a polynomial of a single variable.
2. Distinguishes among linear, quadratic and cubic functions.
3. States the conditions for two polynomials to be identical.

Guidelines to learning - teaching process :

1.
- Introduce the form of a polynomial as $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_n$ where $a_1, a_2, ..., a_n \in \mathbb{R}$ and $n \in \mathbb{Z}_0^+$
- Introduce the terms , degree, leading term and leading coefficient of a polynomial.

2.
- Introduce the general form of a linear function as $f(x) = ax + b$ where $a, b \in \mathbb{R};\ a \neq 0$,
- Introduce the general form of quadratic function as $f(x) = ax^2 + bx + c,\ a,\ b,\ c \in \mathbb{R}$; $a \neq 0$ and
- Introduce the general form of cubic function as $f(x) = ax^3 + bx^2 + cx + d,\ a,\ b,\ c,\ d \in \mathbb{R}$, $a \neq 0$

3.
- States that if $P(x) \equiv Q(x)$, then for all $a \in \mathbb{R}$, $P(a) = Q(a)$ and coefficient of corresponding terms are equal.
- Guide students to use the above property in problem solving.

Competency level 4.2 : Applies algebraic operations to polynomials.

Number of periods : 01

Learning outcomes :
1. Explains the basic Mathematical operations on polynomials.
2. Divides a polynomial by another polynomial.

Guidelines to learning teaching process :

1. Review the prior knowledge relating to addition, substraction and multiplication.
2. - Introduce the notation $\frac{P(x)}{Q(x)}$ for rational polynomials (for $Q(x) \neq 0$)
   - $P(x)$ divided by $Q(x)$ is denoted by $\frac{P(x)}{Q(x)}$ if $P(x) = Q(x) \cdot R(x)$ for some polynominal $R(x)$.
   - Using examples explain the division and long division.

Competency level 4.3 : Solves problems using remainder theorem, factor theorem and its converse.

Number of periods : 05

Learning outcomes :
1. States the algorithm for division.
2. States and proves remainder theorem.
3. Expresses the factor theorem and its converse.
4. Solves problems involving remainder theorem and factor theorem.
5. Defines zeros of a polynomial.
6. Solves the polynomial equations.(up to 4^{th} order)

Guidelines to learning - teaching process :

1. Explain that
   Dividend $\equiv$ Quotient $\times$ Divisor $+$ Remainder
2. - Express that when a polynomial $f(x)$ is divided by $(x-a)$ the remainder is $f(a)$. where a is a constant.
   - states and proves the remainder theorem.

3. - Express that if $f(a)=0$

     $(x-a)$ is a factor of $f(x)$, where a is a constant

   - States the factor theorem.

   - Show that if $(x-a)$ is a factor of $f(x)$ then $f(a)=0$.

   - States the converse of factor theorem

4. - Guide students to solves problems involing remainder theorem and factor theorem (maximum 4 unknowns)

5. - State that in a polynomial $P(x)$, the values of x for which $P(x)=0$ are defined as the zeros of the polynomial.

6. - Guide the students to solves problems Involving polynomials remainder theorem and factor theorem.

Competency 10 : **Manipulates Trigonometric Identities**

Competency level 10.1 : Uses Pythagorean Identities.

Number of periods : 04

Learning outcomes :

1. Explains an trignometric identity.
2. Explains the difference between trignometric identities and trignometric equations.
3. Obtains Pythagorean Identities.
4. Solves problems involving Pythagorean Identities.

Guidelines to learning - teaching process :

1. Introduce an trignometric identity as an equation which is satisfied by every given value of the variable.

2.
   - State that it is not compulsory for an equation to be satisfied by every value of a given variable.
   - Explain by using examples.
     Any identity is an equation; but not all equations need not be an identity.

3. Guide the students to derive Pythagorean trigonometric Identities.

   $\cos^2\theta + \sin^2\theta = 1$

   $1 + \tan^2\theta = \sec^2\theta$

   $1 + \cot^2\theta = \cos ec^2\theta$; for any value of θ

4. Guide students to solve problems involving Pythagorean trigonometric identities.

Competency level 10.2 : Solves trigonometric problems using sum formulae and difference formulae.

Number of periods : 02

Learning outcomes :
1. Constructs addition formulae
2. Uses addition formulae

Guidelines to learning - teaching process :

1. Guide students to obtain the following formulae

   i. $\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$ and deduce the following formulae.

   ii. $\cos(A+B) = \cos A \cdot \cos B - \sin \cdot A \sin B$

   iii. $\sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B$

   iv. $\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B$

   v. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

   vi. $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

2. Explain the methods through examples in using above formulae inorder to solve the trigonometric problems.

Competency level 10.3 : Solves trigonometric problems using product- sum and sum- product formulae.

Number of periods : 05

Learning outcomes :

1. Manipulates product - sum and sum - product formulae.
2. Solves problems involving product - sum and sum - product formulae.

Guidelines to learning - teaching process :

1. Guide students to obtain the following formulae.

   i. $2\sin A\cos B = \sin(A+B)+\sin(A-B)$

   ii. $2\cos A\sin B = \sin(A+B)-\sin(A-B)$

   iii. $2\cos A\cos B = \cos(A+B)+\cos(A-B)$

   iv. $2\sin A\sin B = \cos(A-B)-\cos(A+B)$

   v. $\sin C+\sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

   vi. $\sin C-\sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

   vii. $\cos C+\cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

   viii. $\cos C-\cos D = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)$

2. Direct students to solves trigonometric problems using product - sum and sum - product formulae.

Competency level 10.4 : Solves trigonometric problems using double angles, triple angles and half angles formulae.

Number of periods : 03

Learning outcomes :

1. Derives trigonometric formula for double angle, trible angle and half angle.
2. Solves problems using double angle, triple angle and half angles.

Guidelines to learning - teaching process :

1. i. $\sin 2A = 2\sin A\cos A$

   ii. $\cos 2A = \cos^2 A - \sin^2 A$
   $= 2\cos^2 A - 1$
   $= 1 - 2\sin^2 A$

   iii. $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

   iv. $\sin 3A = 3\sin A - 4\sin^3 A$

   v. $\cos 3A = 4\cos^3 A - 3\cos A$

   - Express $\sin A$, $\cos A$, $\tan A$, in terms of $\tan\dfrac{A}{2}$ as shown above in (i), (ii) and (iii) Express sin, cos, interms of tan.

2. - Guide students to solve problems involving above results.

   - Direct students to prove trignometric identities related to angles of a triangle
   Eg: For any triangle.
   i. When A+B+C= π,
   $\sin(A+B) = \sin(\pi - C) = \sin C$, etc...

   ii. $\dfrac{A+B+C}{2} = \dfrac{\pi}{2}$ show that
   $$\sin\frac{A+B}{2} = \sin\left(\frac{\pi}{2} - \frac{C}{2}\right) = \cos\left(\frac{C}{2}\right)$$

Competency 5 : Resolves rational functions into partial fractions

Competency level 5.1 : Resolves rational functions into partial functions.

Number of periods : 06

Learning outcomes :

1. Defines rational functions.
2. Defines proper rational functions and improper rational functions.
3. Finds partial fractions of proper rational functions. (upto 4 unknowns)
4. Finds partial fractions of improper rational functions. (upto 4 unknowns)

Guidelines to learning - teaching process :

1. • A function of the from $\frac{P(x)}{Q(x)}$, where *P(x)* and *Q(x)* are polynomials in x with $Q(x) \neq 0$ is called a rational function. It is domain is the set of values of x for which $Q(x) \neq 0$

2. • If the degree of the polynomial in the numerator < the degree of the polynomial in the denominator, then the rational function is said to be a proper rational funtion.

   • If the degree of the polynomial in the numerator $\geq$ the degree of the polynomial in the denominator, then the rational function is said to be improper rational funtion.

3. Guide students to resolves rational functions into partial fractions (Maximum 4 unknowns)

   Consider the following cases

   • When $\mathrm{Q}(x)$ can be expressed as linear factors

   • When $\mathrm{Q}(x)$ can be expressed with one or two quadratic factors.

   • When $\mathrm{Q}(x)$ can be expressed with repeated factors.

4. Guide students to resolves improper rational functions into partial fractions (Maximum degree = 4)

Consider the following cases:

- If degree of $P(x)$ = degree of $Q(x)$ then $\dfrac{P(x)}{Q(x)}$ can be written in the form, $\dfrac{P(x)}{Q(x)} = K + \dfrac{R(x)}{Q(x)}$, where degree of $R(x)$ < degree of $Q(x)$ and K is a constant.

- If the degree of P(x) > degree of $Q(x)$ then $\dfrac{P(x)}{Q(x)}$ can be written in the form $\dfrac{P(x)}{Q(x)} = h(x) + \dfrac{R(x)}{Q(x)}$ where degree of $R(x)$ < degree of $Q(x)$, and $h(x)$ is a polynomial called the quotient when $P(x)$ divided by $Q(x)$.

  We have to find $h(x)$ and express $\dfrac{R(x)}{Q(x)}$ into partial fractions.

Competency 6 : Manipulates laws of indices and laws of logarithms.

Competency level 6.1 : Uses laws of indices and laws of logarithms to solve problems.

Number of periods : 01

Learning outcomes :
1. Uses laws of indices.
2. Uses laws of logarithms
3. Uses change of base.

Guidelines to learning - teaching process :

1. Remind the following for $a,\ b \in \mathbb{R}$ and $m, n \in \mathbb{Z}$ as laws of indices

   i. $a^m \times a^n = a^{m+n}$

   ii. $\dfrac{a^m}{a^n} = a^{m-n}$

   iii. $a^{-n} = \dfrac{1}{a^n} = \left(\dfrac{1}{a}\right)^n$ for $a \neq 0$

   iv. $a^0 = 1$ for $a \neq 0$

   v. $\left(a^m\right)^n = a^{mn}$

   vi. $(ab)^m = a^m \times b^m$

   vii. $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$ for $b \neq 0$

n^{th} root of a real number

- Let a and b be real numbers and let $(n \geq 2)$ be an integer. If $b^n = a$ then b is an n^{th} root of a.

  It is a square root when n=2, and it is a cube root when n=3.

- There are two roots if $a > 0$ and n is even. these roots are equal in magnitude and opposite in sign.

- **principal n^{th} root**
  Let a be a real number that has at least one n^{th} root . The principle n^{th} root of a is the n^{th} root that has the same sign as a and it is denoted by $a^{\frac{1}{n}}$ or $\sqrt[n]{a}$.

  (When $n=2$ we omit the index $\boldsymbol{n}$ and write $\sqrt{a}$.)

Let a and b are real numbers such that the indicated roots exist as real numbers, and let $m, n \in \mathbb{Z}^{+}$,
Then,

i. $\sqrt[n]{a^m} = \left(n\sqrt{a}\right)^m$

ii. $\sqrt[n]{a}.\sqrt[n]{b} = \left(\sqrt[n]{ab}\right)$

iii. $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \left(\sqrt[n]{\dfrac{a}{b}}\right)$

iv. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

v. $\left(\sqrt[n]{a}\right)^n = |a|$ where n is even

vi. $\sqrt[n]{a^n} = a$ where n is odd

- Explain the above results with examples.
- Solves problems involving indices.

2. Using laws of indices , define logrithms as
$a^b = N \Leftrightarrow b = \log_a N, \quad (a \neq 1, a > 0, N > 0)$

Laws of logarithms

- $\log_a (MN) = \log_a M + \log_a N$
- $\log_a \left(\dfrac{M}{N}\right) = \log_a M - \log_a N$
- $\log_a N^p = p \log_a N,\ P \in Q$ and $a, M, N \in \mathbb{R}^{+}$

3. **Change of base.**

- $\log_a b = \dfrac{1}{\log_b a}$, where $a, b > 0$
- $\log_a b = \dfrac{\log_c b}{\log_c a}$, where $a, b, c > 0$

Competency 7 : Solves inequalities involving real numbers

Competency level 7.1 : States basic properties of inequalites

Number of periods : 04

Learning outcomes :
1. Defines inequalities
2. States the Trichotomy law.
3. Represents inequalities on a real number line.
4. Denotes inequalities in terms of interval notation.

Guidelines to learning - teaching process :

Note that if a is positive then $a-0=a\in\mathbb{R}^+$
Therfore if a is positive, then a > 0

1. • When a and b are real numbers,
   i. $a>b$ only if *(a-b)* is positive
      $a > b$ only if $(a - b) > 0$
   ii. $a<b$ only if $(a-b)$ is negative
      $a < b$ only if $(a - b) < 0$

2. • When x and y are any two real numbers exactly one of the following is true:
   $x>y$, $x<y$, $x=y$

3. • Explain inequalities using number line.

4. • Introduce the following interval notations for a set of numbers.
   When $a,\ b\in\mathbb{R},\ \ a<b,$

Interval	Notation
$\{x\in\mathbb{R}\ \ a\le x\le b\}$	$[a,\ b]$
$\{x\in\mathbb{R}\ \ a\le x<b\}$	$[a,\ b)$
$\{x\in\mathbb{R}\ \ a<x\le b\}$	$(a,\ b]$
$\{x\in\mathbb{R}\ \ a<x<b\}$	$(a,\ b)$

Explain following intervals as well.

$\{x\in\mathbb{R}\ \ x\ge a\}$	$[a,\ +\infty)$
$\{x\in\mathbb{R}\ \ x>a\}$	$(a,\ +\infty)$
$\{x\in\mathbb{R}\ \ x\le a\}$	$(-\infty,\ a]$
$\{x\in\mathbb{R}\ \ x<a\}$	$(-\infty,\ a)$

Competency level 7.2 : Analyses inequalities.

Number of periods : 04

Learning outcomes :
1. States and proves fundamental results on inequalities.
2. Solves inequalities involving algebraic expressions.
3. Solves inequalities including rational functions, algebraically and graphically.

Guidelines to learning - teaching process :

1. Results.

   When $a, b, c \in \mathbb{R}$

   i. $a > b$ and $b > c \Rightarrow a > c$

   ii. $a > b \Rightarrow a + c > b + c$

   iii. $a > b$ and $c > 0 \Rightarrow ac > bc$

   iv. $a > b > 0$ and $c < 0 \Rightarrow ac < bc$

   v. $a > b$ and $c = 0 \Rightarrow ac = bc = 0$

   vi. $a > b$ and $c > d \Rightarrow a + c > b + d$

   vii. $a > b > 0$ and $c > d > 0 \Rightarrow ac > bd$

   viii. $a > b > 0 \Rightarrow \frac{1}{a} < \frac{1}{b}$

   ix. $a < b < 0 \Rightarrow \frac{1}{a} > \frac{1}{b}$

   x. For $a > b > 0$ and n is a positive rational number, $a^n > b^n$ and $a^{-n} < b^{-n}$

2. When $f(x)$ and $g(x)$ are two function of x (linear or quadratic)solve the inequalities of the form,

   $f(x) \geq g(x)$, $f(x) > g(x)$, $f(x) \leq g(x)$, $f(x) < g(x)$.

   - Guide students to find solutions using algebraic or graphical methods.

3. Consider a rational function of the form $\frac{P(x)}{Q(x)}$ where $P(x)$, $Q(x)$ are polynomcals in x and their degree are len than or equal to 2 (only algebraic method).

Competency level 7.3 : Solves inequalities involving modulus (absolute value) function.

Number of periods : 06

Learning outcomes :
1. States the modulus (absolute value) of a real number.
2. Sketches the graphs involving modulus functions.
3. Solves inequalities involving modulus. (only for linear functions)

Guidelines to learning - teaching process :

1. Inequalities including modulus

   Let $x \in \mathbb{R}$

   Define $|x| = \begin{cases} x \ ; \ x \geq 0 \\ -x \ ; \ x < 0 \end{cases}$

2. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function

   $|f|$ is defined as follows.

   $|f| : \mathbb{R} \rightarrow \mathbb{R}$

   $|f|(x) \ = \ |f(x)|$, where

   $|f(x)| = \begin{cases} f(x) \ ; \ f(x) \geq 0 \\ -f(x) \ ; \ f(x) < 0 \end{cases}$

   Illustrate with examples.

   - Draw the graphs of modulus functions.

   - Direct students to draw graphs of the functions such as

   $y = |ax|, \quad y = |ax + b|, \ \ y = |ax| + b$

   $y \ = \ |ax + b| + c$

   $y \ = \ c - |ax + b|$

   $y \ = \ |ax + b| \pm |cx + d|$

   $y \ = \ |ax^2 + bx + c|$

   where $a, b, c, d \in \mathbb{R}$.

3. Determine the solution set of inequalities such as

   $|ax + b| \ \geq \ |cx + d|$

   $|ax + b| \ \geq \ lx + m$

   $|ax + b| \ \pm \ |cx + d| \ \geq \ k$

   i. algebraically ii. graphically

   where $a, b, c, d, k \in \mathbb{R}$ and k is a constant.

Competency 9 : Interprets circular functions (Trigonometric functions)

Competency level 9.5 : Finds general solutions

Number of periods : 04

Learning outcomes : 1. Solves trigonometric equations

Guidelines to learning - teaching process :

1. - General solutions
     If $\sin\theta = \sin\alpha$, then $\theta = n\pi + (-1)^n \alpha$, where $n \in \mathbb{Z}$
     If $\cos\theta = \cos\alpha$ then $\theta = 2n\pi \pm \alpha$, where $n \in \mathbb{Z}$
     If $\tan\theta = \tan\alpha$ then $\theta = n\pi + \alpha$, where $n \in \mathbb{Z}$
     Solves trignimetic

   - Equations that can be solved by factoring.

   - Equations that can be solved using Pythagorean identities, addition formulae and multiplication formulae.

   - Equations that can be solved by using the formulae of double angles, triple angles and half angles.

   - Solutions of equations that can be converted to the above forms are also expected.

   - Equations of the form
     $a\cos\theta + b\sin\theta = c$, where $c \leq \sqrt{a^2 + b^2}$

Combined Mathematics II

Competency 1 : Manipulates Vectors

Competency level 1.1 : Investigates vectors

Number of periods : 03

Learning outcomes :

1. States the differnece between scalar quantities and scalars.
2. Explains the difference between vector quantity and a vector.
3. Represents a vector geometrically.
4. Expresses the algebraic notation of a vector quantity.
5. Defines the modulus of a vector.
6. Defines a "null vector"
7. Defines $-a$ where a is a vector
8. States the conditions for two vectors to be equal.
9. States the triangle law of addition of two vectors.
10. Deduces the parallelogram law of addition for two vectors.
11. Adds three or more vectors.
12. Multiplies a vector by a scalar.
13. Substracts a vector from another.
14. Identifies the angle between two vectors.
15. Identifies parallel vectors.
16. States the conditions for two vectors to be parallel.
17. Defines "unit vector".
18. Resolves a vector in given directions.

Guidelines to learning - teaching process :

1. State that a quantity having only magnitude and expressed using a certain measuring unit is called a scalar quantity and that numerical value without unit is a scalar.

2. Explain vector quantities as those with magnitude, direction measuring units and obeying triangle law of addition [Triangle law of addition will be given later] and without units is a vector.

3.
   - Explain that a line segment with a magnitude and a direction is geometrically known as a vector.
   - A vector does not have dimensions although a vector quantity has dimensions.

4. • Present that the vector represented by the line segment AB from A to B is denoted by $\overrightarrow{\mathrm{AB}}$.

• Show that the " vector a" is denoted by the symbol $\underline{a}$ or ***a*** (In print, letters in bold print) are used to denote vectors. State also that different letters are used to denote different vectors.

5. Introduce the magnitude of a vector as its modulus, show that the modulus of $\underline{a}$ is denoted by $|\underline{a}|$. Explain that $|\underline{a}|$ is always a non negative scalar, because it is the length of a line segment.

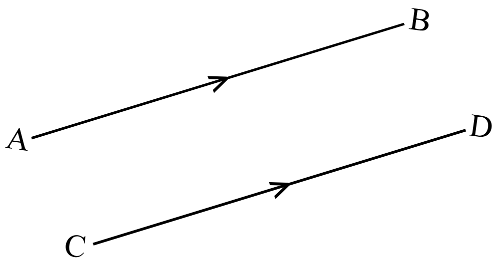
6. Define a vector of zero magnitude in any given direction as the null vector. It is denoted by $\underline{0}$.

   Further explain the statements $\underline{a}+(-\underline{a})=\underline{0}$ and $\overrightarrow{\mathrm{AA}}=\underline{0}$

   State also that this is read as 'vector 0'

7. Define the vector equal in magnitude and opposite in direction to the vector $\underline{a}$ as the reversed vector of $\underline{a}$ and denote it by $-\underline{a}$.

8. Vectors with equal magnitude and in the same direction are called equal vectors.



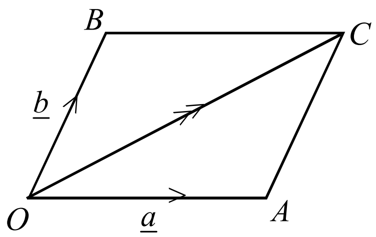
   When the two vectors $\underline{a}$ and $\underline{b}$ are geometrically represented by $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{CD}}$.

$$\underline{a}=\underline{b} \Leftrightarrow \begin{cases} \mathrm{AB}=\mathrm{CD} \\ \mathrm{AB} \parallel \mathrm{CD} \\ \text{Direction of } \overrightarrow{\mathrm{AB}} \text{ must be same as} \end{cases}$$
$$\text{the direction of } \overrightarrow{\mathrm{CD}}.$$

9. Triangle law of addition of vectors:-

   If $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{BC}}$ represent the two vectors $\underline{a}$ and $\underline{b}$ respectively, then $\overrightarrow{\mathrm{AC}}$ represents $(\underline{a}+\underline{b})$. Show that addition of two vectors results in a vector. (Closure property).

10. Using the above triangle law of addition of vectors, deduce the parallelogram law of addition of two vectors.



$Let\ \overrightarrow{OA} = \underline{a}\ and\ \overrightarrow{OB} = \underline{b}$

$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = (triangular\ law)$

$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{OB}\ \left\{\overrightarrow{AC} = \overrightarrow{OB}\ equal\ \ vectors\right\}$

$\therefore\ \ \overrightarrow{OC} = \underline{a} + \underline{b}$

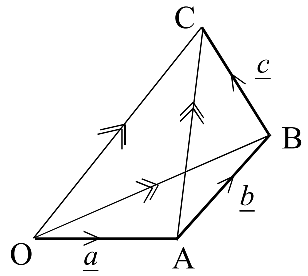
11. Show how three or more vectors are added using the law of addition for two vectors, repeatedly.

$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$

$\underline{a} + \underline{b} = \overrightarrow{OB}$

$\overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{OC}$

$\left(\underline{a} + \underline{b}\right) + \underline{c} = \overrightarrow{OC}$



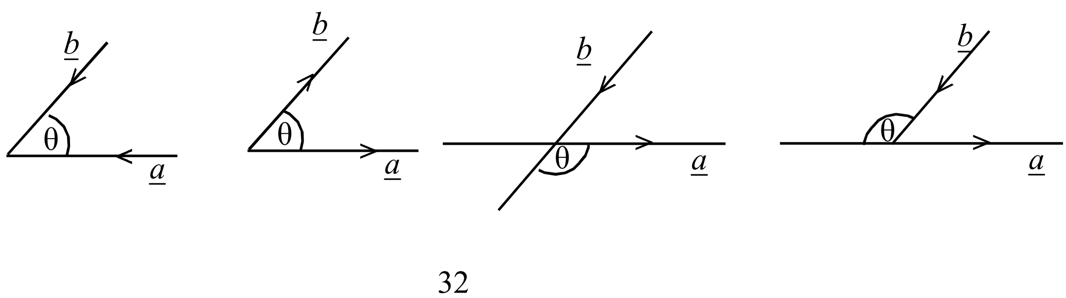
12. When $\underline{a}$ is a vector and k is a scalar, introduce that $k\underline{a}$ as k times $\underline{a}$. Discuss the cases when $k > 0$, $k = 0$ and $k < 0$. Give examples. Describe the vector $k\underline{a}$.

13. State that the subtraction of vector $\underline{b}$ from $\underline{a}$ is same as the addition of vector $\left(-\underline{b}\right)$ to vector $\underline{a}$.

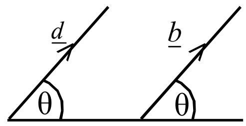
$\underline{a} - \underline{b} = \underline{a} + \left(-\underline{b}\right)$.

Note: Addition and subtraction can do only with to vectors of the same 1type.

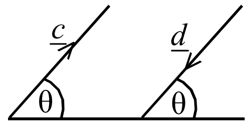
14. Introduce the angle between two vectors as the angle θ, between their directions. $0 \leq \theta \leq \pi$



15. State that vectors whose directions are parallel are called "parallel vectors".



Here $\underline{a}$ and $\underline{b}$ are parallel vectors.



Here $\underline{c}$ and $\underline{d}$ are parallel vectors.

16. Show that $\underline{a}$ and $\underline{b}$ are parallel, if $\underline{b} = k\underline{a}$ where k is a non zero scalar.

17.
   - Define a vector of unit magnitude as a unit vector.
   - If $\underline{a}$ is a unit vector then $|\underline{a}| = 1$.
   - If $\underline{a}$ is a given non zero vector and $\underline{u}$ is a unit vector in direction $\underline{a}$ then, $\underline{a} = |\underline{a}|\underline{u}$ then $\underline{u} = \dfrac{\underline{a}}{|\underline{a}|}$

18.
   - Show how a given vector can be resolved in two given directions by constructing a parallelogram with given vector as a diagonal and adjacent sides in the required directions.
   - Show how a given vector can be resolved in two given mutually perpendicular directions by constructing a rectangle with the given vector as a diagonal.

Competency level 1.2 : Constructs algebraic system for vectors.

Number of periods : 01

Learning outcomes : 1. States the properties of addition and multiplication by a scalar.

Guidelines to learning - teaching process :

1. State and prove the following properties of addition of vectors.
   - i. Commutative law; If $\underline{a}$ and $\underline{b}$ are two vectors then $\underline{a} + \underline{b} = \underline{b} + \underline{a}$.
   - ii. Associative law:- When $\underline{a}$, $\underline{b}$ and $\underline{c}$ are three vectors. $(\underline{a} + \underline{b}) + \underline{c} = \underline{a} + (\underline{b} + \underline{c})$
   - iii. Distributive law:- When h and k are scalars. $k(\underline{a} + \underline{b}) = k\underline{a} + k\underline{b}$ and $(h + k)\underline{a} = h\underline{a} + k\underline{a}$.

Competency level 1.3 : Applies position vectors to solve problems

Number of periods : 06

Learning outcomes :

1. Defines position vectors.
2. Expresses the position vector of a point in terms of the cartesian coordinates of that point.
3. Adds and subtracts vectors in the form $x\underline{i} + y\underline{j}$
4. Proves that if $\underline{a}$, $\underline{b}$ are two non zero, non parallel vectors and if $\lambda\underline{a} + \mu\underline{b} = \underline{0}$ then $\lambda = 0$ and $\mu = 0$ where λ, μ are scalars
5. Solves probelms involving these results

Guidelines to learning - teaching process :

1. Define the positions vector of a point P with reference to an origin O as vector OP or $\overrightarrow{OP} = \underline{r}$.

2. Introduce the unit vectors $\underline{i}$ and $\underline{j}$
   - If the component of a vector along OX is *x*, and the component along OY is *y*, then show that the vector can be expressed in the form $x\underline{i} + y\underline{j}$.
   - Show that, related to origin O, P is the point in two dimension. Let P $\equiv (x, y)$, then position vector of P related to O denoted by $\underline{r}$ and $\underline{r} = x\underline{i} + y\underline{j}$
   - Show also that $|\underline{r}| = \sqrt{x^2 + y^2}$ in two dimensions.

3. If $\underline{a}_1 = x_1\underline{i} + y_1\underline{j}$ and $\underline{a}_2 = x_2\underline{i} + y_2\underline{j}$, show that $\underline{a}_1$ and $\underline{a}_2$ can be added and subtracted as follows.

   $\underline{a}_1 + \underline{a}_2 = (x_1 + x_2)\underline{i} + (y_1 + y_2)\underline{j}$

   $\underline{a}_1 - \underline{a}_2 = (x_1 - x_2)\underline{i} + (y_1 - y_2)\underline{j}$

   - Show that this is also applicable to more than two vectors

4. Direct students to prove that if $\underline{a}$, $\underline{b}$ are two given non zero, non parallel vectors then if $\lambda\underline{a} + \mu\underline{b} = \underline{0} \Leftrightarrow \lambda = 0$ and $\mu = 0$.

5. Guide students to solves problems involving above result.

Competency level 1.4 : Interprets scalar and vector product.

Number of periods : 04

Learning outcomes :

1. Defines the scalar product of two vectors.
2. States that the scalar product of two vectors is a scalar.
3. States the properties of scalar product.
4. Finds the angle between two non zero vectors.
5. Explain condition for two non zero vectors to be perpendicular to each other.
6. Defines vector product of two vectors.
7. States the properties of vector product . (Application of vector product are not expected)

Guidelines to learning - teaching process :

1. Scalar product (dot product)

   If $\underline{a}$ and $\underline{b}$ are any two non zero vectors and the angle between them is $\theta(0 \le \theta \le \pi)$, then their scalar product defined $\underline{a} \cdot \underline{b} = |\underline{a}||\underline{b}|\cos\theta$. State that this is also known as the dot product. If $\underline{a} = \underline{0}$ or $\underline{b} = \underline{0}$ then $\underline{a} \cdot \underline{b} = 0$.

2. Explain that $|\underline{a}||\underline{b}|\cos\theta$ is a scalar.
   - Show that $\underline{a} \cdot \underline{b} = 0$ if $\underline{a} \perp \underline{b}$ and
   - $(\underline{a} \cdot \underline{a}) = |\underline{a}|^2 = a^2$, where $a = |\underline{a}|$

3. i. Commutative law $\underline{a} \cdot \underline{b} = \underline{b} \cdot \underline{a}$

   ii. Distributive law $\underline{a}.(\underline{b} + \underline{c}) = \underline{a} \cdot \underline{b} + \underline{a} \cdot \underline{c}$

4. If $\underline{a}$ and $\underline{b}$ are any two non zero vectors if θ is the angle between them then,

   $$\cos\theta = \frac{\underline{a}.\underline{b}}{|\underline{a}| \cdot |\underline{b}|}$$

5. If $\underline{a} \perp \underline{b}$ then $\theta = \frac{\pi}{2}$

   $\Rightarrow \cos\theta = 0$

   $\Rightarrow \underline{a}.\ \underline{b} = 0$

6. Vector Product (definition)

- Present the definition that if $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are any two non zero vectors and $\theta(0<\theta<\pi)$ is the angle between them the vector product of $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is defined as $\underset{\sim}{a}\times\underset{\sim}{b}=\left(|\underset{\sim}{a}||\underset{\sim}{b}|\sin\theta\right)\underset{\sim}{n}$.

- Where $\underset{\sim}{n}$ is a unit vector in the direction perpendicular to both $\underset{\sim}{a}$ and $\underset{\sim}{b}$. $(\underset{\sim}{a},\ \underset{\sim}{b},\ \underset{\sim}{n})$ forms a right handed set.

- If $\underset{\sim}{a}=\underset{\sim}{0}$ or $\underset{\sim}{b}=\underset{\sim}{0}$ or $\underset{\sim}{a}\,//\,\underset{\sim}{b}$, then $\underset{\sim}{a}\times\underset{\sim}{b}$ is a null vector.

7. Properties of vector product

- $\underset{\sim}{a}\times\underset{\sim}{b}=-\underset{\sim}{b}\times\underset{\sim}{a}$
- Discus the area interpretation of $|\underset{\sim}{a}\times\underset{\sim}{b}|$

Note: Problems involving vector product are not expected.

Competency 2 : Uses systems of Coplanar forces

Competency level 2.1 : Explains forces acting on a particle.

Number of periods : 02

Learning outcomes :

1. Describes the concept of a particle.
2. Describes the concept of a force.
3. States that a force is a localized vector.
4. Represents a force geometrically.
5. Introduces different types of forces in mechanics.
6. Describes the resultant of a system of coplanar forces acting at a point.

Guidelines to learning - teaching process :

1. State that a particle is considered as a solid body having very small dimensions compared with other distances related to its motion.
   - State that a particle can be considered as a sphere of zero radius having mass, it can be represented geometrically by a point.

2. State force as an action which creates a motion in a body at rest or which changes the nature of the motion in the case of a moving body.

3. A force has a magnitude, point of action and a line of action. Therefore it can be treated as a localized vector.

4. State that Newton (N) is the unit by which the magnitude of a force is measured.Show that a force can be represneted by a line segment whose length is proportional to the magnitude of the force and drawn in its direction.

5. Different types of forces.
   - i. Force of attraction: Weight of a body.
   - ii. Normal reactions between surfaces touching each other.
   - iii. Reaction between rough surfaces in contact. (the normal reaction acts along the common normal at the point of contact and the frictional force acts along a common tangent).
   - iv. Tension in a string.
   - v. Thrusts or tensions in light rods.
   - State that tension and thrust are also known as stresses.

6. Introduce the "resultant" of two or more forces acting at a paticle as the single force whose effect is equivalent to that of all the forces acting at the particle.

Competency level 2.2 : Explains the action of two forces acting on a particle.

Number of periods : 04

Learning outcomes :
1. States resultant of two forces.
2. States the parallelogram law of forces.
3. Uses the parallelogram law to obtain formulae to determine the resultant of two forces acting at a point.
4. Solves problems using the parallelogram law of forces.
5. Writes the condition necessary for a particle be in equilibrium under the action of two forces.
6. Resolves a given force into two components in two given directions.
7. Resolves a given force into two components perpendicular to each other.

Guidelines to learning - teaching process :

1. • Two froces acting on a particle in same direction.
   • Two froces acting on a particle in opposite direction.

2. Introduce the "Parallelogram law of forces" to find the resultant of two forces acting on a particle.

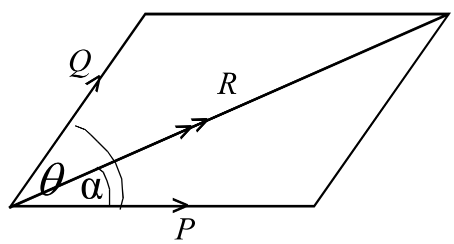
   Parallelogram law of forces.

   If two forces acting at a point can be represented in magnitude and direction by the two adjacent sides of a parallelogram drawn with the given point as a vertex, then the diagonal through that vertex represents the resultant of the two forces in magnitude and direction.

3. Show that $R^2 = P^2 + Q^2 + 2PQ\cos\theta$ and

   $$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$

   Gives the magnitude R and the angles α between $\underline{R}$ and $\underline{P}$



Discuss specially the cases where,

(i) $P = Q$ (ii) $P \perp Q$

(iii) $\theta = 0$ (iv) $\theta = \pi$

Note that when $P = Q$ the resultant bisect the angle between the forces.

4. Direct students to solve problems involving two forces acting on a particle.

5. If the two forces acting on a particle are equal in magnitude opposite in direction, then that particle is said to be in equilibrium under the action of two forces.

6. Show the method of resolving a given force in two directions by constructing a parallelogram with its adjacent sides in the two given directions and its diagonal representing the given force.

   Show that the effect of the two resolved parts is same as the effect of the given force.

7. State that a force is resolved into two components perpendicular to each other for the convenience in solving problems and obtain the two components.

Competency level 2.3 : Explains the action of a system of coplanar forces acting on a particle.

Number of periods : 04

Learning outcomes :
1. Determines the resultant of three or more coplanar forces acting at a point by resolution.
2. Determines graphically the resultant of three or more coplanar forces acting at a particle.
3. States the conditions for a system of coplanar forces acting on a particle to be in equilibrium.
4. Writes the condition for eqilibrium
5. Completes a polygon of forces.

Guidelines to learning - teaching process :

1. Show that, if X and Y are the algebraic sums of the components of the forces when resolved in two given perpendicular directions then their resultant is also given by $R = \sqrt{X^2 + Y^2}$ and the angle α which the resultant makes with the direction of X is given by $\tan\alpha = \frac{Y}{X}$

   - Direct students to solve problems using these results.

2. Present the graphical method (polygon method) of determining the resultant of three or more coplanar forces acting at a point.

3. Null resultant vector $\underline{R} = X\,\underline{i} + Y\,\underline{j} = \underline{0}$
   - Completion of polygon of forces.

4. The algebraic sums of the resolved components in two given perpendicular directions are zero. i.e. $X = 0, Y = 0$

   $\underline{R} = X\underline{i} + Y\underline{j} = \underline{0}$

   $\Leftrightarrow X = 0,\ Y = 0$

5. Guide students to complets a polygon of forces.

Second Term

Combined Mathematics I

Competency 3 : Analyses Quadratic Functions.

Competency level 3.1 : Explores the properties of quadratic functions.

Number of periods : 10

Learning outcomes :
1. Introduces quadratic functions.
2. Explains what a quadratic function is.
3. Explains the properties of a quadratic function.
4. Sketches the graph of a quadratic function.
5. Describes the different types of graphs of the quadratic function
6. Describes zeros of quadradic functions.

Guidelines to learning - teaching process :

1. Recall that the function $f(x) = ax^2 + bx + c$ is called a quadratic function when $a \neq 0$ and $a,\ b,\ c \in \mathbb{R}$.

2. Explains that "what is quadraic function"

3. Guide to express the quadratic function can be written in the form $f(x) = a(x+p)^2 + q,\ \ a,\ p,\ q \in \mathbb{R},\ \ a \neq 0$.

   Where $p = \dfrac{b}{2a},\ q = \dfrac{4ac - b^2}{4a}$

   - Discuss the sign of the quadratic function for the values of x.
   - Discuss the symetry and explain that the graph of the function is symmetrical about the line $x = -p$.
   - Discuss the behaviour of the quadratic function,

     (i) When $\Delta > 0,\ \ a > 0$ and $a < 0$

     (ii) When $\Delta = 0,\ \ a > 0$ and $a < 0$

     (iii) When $\Delta < 0,\ \ a > 0$ and $a < 0$

     Where $\Delta = b^2 - 4ac$ is called the discriminant of the function $f(x) = ax^2 + bx + c$.

     Express $f(x) = a\left\{\left(x + \dfrac{b}{2a}\right)^2\right\} - \left(\dfrac{b^2 - 4ac}{a}\right)$

     Explain that where *a > 0 f(x)* has a local minimum when *a<0 f(x)* has a local maximum

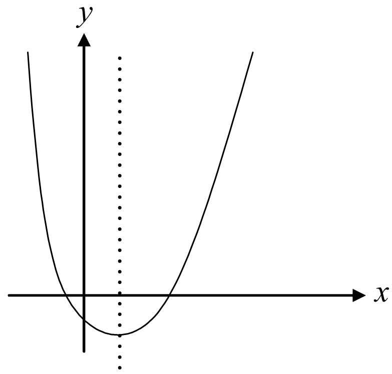
4. Direct students to draw the graphs of quadratic functions for the cases $b^2 - 4ac > 0$, $b^2 - 4ac < 0$ and $b^2 - 4ac = 0$ in each cases consider $a > 0$ and $a < 0$

- Emphasises the properties of the quadratic function by means of the graphs drawn by students.

5.

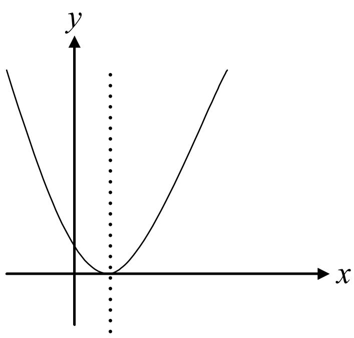
$b^2 - 4ac > 0$

$a > 0$



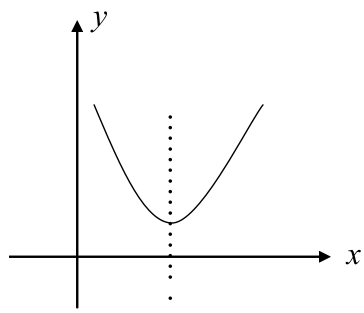
$b^2 - 4ac = 0$

$a > 0$



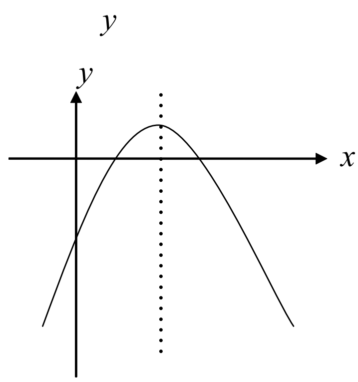
$b^2 - 4ac < 0$

$a > 0$

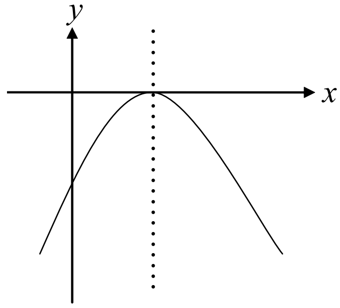


$b^2 - 4ac > 0$

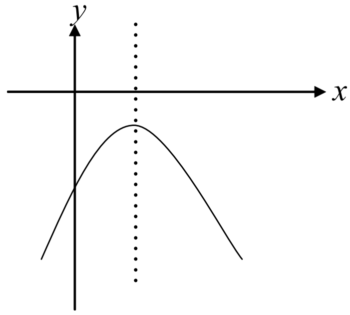
$a < 0$



$b^2 - 4ac = 0$
$a < 0$



$b^2 - 4ac < 0$
$a < 0$



6. • Describes zero of a quadratic function
   • Solves problems involving quadratic function

Competency level 3.2 : Interprets the roots of a quadratic equation.

Number of periods : 15

Learning outcomes :
1. Explains the roots of a quadratic equation.
2. Finds the roots of a quardratic equation.
3. Expresses the sum and product of the roots of a quadratic equation in terms of its coefficients.
4. Describes the nature of the roots of a quardatic equation.
5. Finds quadratic equations whose roots are symmetric expressions of α and β.
6. Solves problems involving quadratic functions and quadratic equations.
7. Transforms roots to other forms

Guidelines to learning - teaching process :

1. State that when $a \neq 0$, $a, b, c \in \mathbb{R}$, $ax^2 + bx + c = 0$ which gives the zero points of the quadratic function is called the roots of the quadratic equation.

2. Prove that a quardatic equation cannot have more than two distinct roots.

   Prove that a quardatic equation in a single variable can have only two roots.

   If roots are α and β then show that

   $$\alpha = \frac{-b-\sqrt{b^2-4ac}}{2a} \ , \ \beta = \frac{-b+\sqrt{b^2-4ac}}{2a}$$

3. If the roots of the quardatic equation $ax^2+bx+c=0$ are α and β then show that $\alpha+\beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$.

4. Accordingly as $b^2-4ac>0$, $b^2-4ac=0$ and $b^2-4ac<0$, show that the roots of the quardatic equation are real and distinct or real and coincident or (imaginary) non real

   Show the converse is also true.

   - Explain that the necessary and sufficient condition for the roots of the quardatic equation to be real is

     $b^2-4ac \geq 0$

     $\Delta = b^2-4ac$ is called the discriminant of the equation

     $ax^2+bx+c=0$

   - Obtain the occassions, both roots are positive, both roots are negative one is positive and other is negative and one root is zero, uisng the coefficient of the quardratic equation.

5. If the roots of the equation $ax^2+bx+c=0$ are α and β obtain equations of whose roots are symmetric expressions of α and β.

   Eg: (i) α^2, β^2

   (ii) α^3+1, β^3+1

   (iii) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ etc..

   - Discuss the condition of two quadratic equations to have a common root.

6. Direct students to solve problems involving quadratic equations.

7. Use suitable transformation to find equation with symmetriccal expressions..

Competency 12 : Solves problems involving inverse trigonometric functions.

Competency level 12.1 : Describes inverse trignometric functions.

Number of periods : 02

Learning outcomes :
1. Defines inverse trignometric functions.
2. States the domain and the range of inverse trigonometric functions.

Guidelines to learning - teaching process :

1. Explain that if $y = \sin x$, the value of x when y is given, is stated as $x = \sin^{-1} y$ and that $x = \sin^{-1} y$ is not a function. But it can be made a function by limiting the domain of $y = \sin x$.
2.
   - Here the domain of $\sin x$ in generally limited to $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. By interchanging x and y it can be denoted by $y = \sin^{-1} x$ where $-1 \leq x \leq 1$ $-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}$.
   - Explain similarly that $y = \cos^{-1} x$ is definded such that $0 \leq \cos^{-1} x \leq$
   - Define, also, $y = \tan^{-1} x$ and explain that the values included in the domain $-\frac{\pi}{2} < x < \frac{\pi}{2}$ are its principle values.
   - $y = \sin^{-1} x$; Domain [-1, 1], Range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

     $y = \cos^{-1} x$; Domain [-1, 1], Range $[0, \pi]$

     $y = \tan^{-1} x$;Domain $(-\infty, \infty)$, Range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Competency level 12.2 : Represents inverse function graphically.

Number of periods : 02

Learning outcome : 1. Draws the graph of inverse trigonometric functions.

Guidelines to learning - teaching process :

1. Draw the graphs of the following functions:

   $y = \sin^{-1} x$, $y = \cos^{-1} x$, $y = \tan^{-1} x$

   States their domain and range.

Competency level 12.3 : Solves simple problems involving inverse trigonometric functions.

Number of periods : 04

Learning outcomes : 1. Solves simple problems involving inverse trigonometric functions.

Guidelines to learning teaching process :

1. Direct students to solve problems involving inverse trigonometric functions.

Competency 11 : Applies sine rule and cosine rule to solve trigonometric problems.

Competency level 11.2 : Applies sine rule and cosine rule.

Number of periods : 06

Learning outcomes :
1. Proves sine rule.
2. Proves cosine rule.
3. Solves problems involving triangles using sine and cosine rules.

Guidelines to learning - teaching process :

1. Proof of the sine rule, guide students to prove it for all three triangles.

   $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

2. Proof of the cosine rule, guide students to prove it for three triangles.

   $a^2 = b^2 + c^2 - 2bc\cos A$

   $b^2 = a^2 + c^2 - 2ac\cos B$

   $c^2 = a^2 + b^2 - 2ab\cos C$ for all triangles

3.
   - Direct students to solve problems including determination of magnitude of angles or lengths of sides when sufficient data is provided.

   - Guide students to solve problems involving the properties of triangles.

Competency 13 : Determines the limit of a function.

Competency level 13.1 : 1. Explains the limit of a function.

Number of periods : 02

Learning outcomes :
1. Explains the meaning of limit.
2. Distinguishes the cases where the limit of a function does not exist.

Guidelines to learning teaching process :

1. When $x \in \mathbb{R}$ discuss how the value of *x*, can approach a rational number "a" without being equal to it.
2. Explains cases where $\lim_{x \to a} f(x)$ does not exist and distinguish between the limit of a function at a point and the value of a function at that point by examples. Explain graphically too.

Competency level 13.2 : Solves problems using the theorems on limits.

Number of periods : 03

Learning outcome : 1. Expresses the theorems on limits.

Guidelines to learning - teaching process :

1. Assume *f* and *g* to be functions for which a limit exist as $x \to a$, where "*a*" is a real number.
   1. let $f(x) = k$ when *k* is a constant, $\lim_{x \to a} f(x) = k$.
   2. When *k* is a constant, $\lim_{x \to a} k\, f(x) = k \lim_{x \to a} f(x)$.
   3. When *k* is a constant, $f(x) = k \ \lim_{x \to a} f(x) = k$.
   4. $\lim_{x \to a} \left[f(x) \pm g(x) \right] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$.
   5. $\lim_{x \to a} \left[f(x) \cdot g(x) \right] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$.
   6. If $\lim_{x \to a} g(x) \neq 0$, $\lim_{x \to a} \left[\dfrac{f(x)}{g(x)} \right] = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$

7. $\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n$, $n \in \mathbb{N}$

8. $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$, $n \in \mathbb{N}$

   When $f(x) \geq 0$

9. When *f* is a polynomial function for all $x \in \mathbb{R}$

   $\lim_{x \to a} f(x) = f(a)$.

10. If $f(x) = g(x)$ for all values of x except at $x = a$ in an interval including *a*, then

    $\lim_{x \to a} f(x) = \lim_{x \to a} g(x)$.

    The proofs of the above theorems are not expected. Explain their use in solving problems with examples.

Competency level 13.3 : Uses the limit: $\lim_{x \to a}\left(\frac{x^n - a^n}{x - a}\right) = na^{n-1}$ to solve problems.

Number of periods : 03

Learning outcomes :

1. Proves $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$ where n is a rational number.
2. Solves problems involving above ersult.

Guidelines to learning - teaching process :

1. Prove the theorem for positive integral values of *n*, and deduce it for negative integral values of *n*.
   Then prove that the theorem is true for any rational number *n*.
2. Direct students to solve suitable problems.

Competency level 13.4 : Uses the $\lim_{x \to 0}\left(\frac{\sin x}{x}\right) = 1$ to solve problems.

Number of periods : 03

Learning outcomes :

1. States the sandwich theorem. (squeezes lemma)
2. Proves that $\lim_{x \to 0} \frac{\sin x}{x} = 1$
3. Solves the problems using the above result.

Guidelines to learning - teaching process :

1. In an open interval containing a if for all values of x including or excluding a, $f(x) \le h(x) \le g(x)$ and $\lim_{x \to a} f(x) = l = \lim_{x \to a} g(x)$ then $\lim_{x \to a} h(x) = l$.
   The proof of this theorem is not necessary.

2. State that $\lim_{x \to 0} \frac{\sin x}{x} = 1$ (x is measured in radians)
   When x is measured in radians proves that
   $\lim_{x \to 0^+} \frac{\sin x}{x} = 1$ by geometrical method.
   Using the above results deduce that $\lim_{x \to 0^-} \frac{\sin x}{x} = 1$

3. Direct students to solve suitable problems.

Competency level 13.5 : Interprets one sided limits.

Number of periods : 02

Learning outcomes :
1. Interprets one sided limits.
2. Finds one sided limits of a given function at a given real number.

Guidelines to learning - teaching process :

1. Discuss $\lim_{x \to a^+} f(x)$, $\lim_{x \to a^-} f(x)$

2. Convince the students that $\lim_{x \to 0^+} \frac{1}{x} = \infty$ and $\lim_{x \to 0^-} \frac{1}{x} = -\infty$ using the graph. Consider the domain as $\mathbb{R} \setminus \{0\} = \mathbb{R}^+ \cup \mathbb{R}^-$

3. Limits such as when $x \to a^-, f(x) \to \pm\infty$ and $x \to a^+, f(x) \to \pm\infty$ are called infinite limits and are also known as one side limits (left or right).
   recognize the cases
   When $x \to \pm\infty$ the limit of $f(x)$ may be finite or infinite

Competency level 13.6 : Find limits at infinity and its application to find limit of rational functions.

Number of periods : 02

Learning outcomes :
1. Distinguishes the cases where a limit for $f\left(x\right)$ either exist or not as x approaches to infinity.
2. Explains horizontal asymptotes

Guidelines to learning teaching process :

1. In $\lim_{x\to-\infty}\frac{P\left(x\right)}{Q\left(x\right)}$ and $\lim_{x\to+\infty}\frac{P\left(x\right)}{Q\left(x\right)}$; $\left[Q\left(x\right)\neq 0\right]$ where $P\left(x\right)$ and $Q\left(x\right)$ are polynomials and the degree of $P\left(x\right)$ is n and that of $Q\left(x\right)$ is m.

   Discuss seperately the cases when

   (i) $n<m$ (ii) $n=m$ (iii) $n>m$

   with suitable examples.

   State that these are known as limits at infinity.

2. In horizontal asymptote ocurse when

   $\lim_{x\to\pm\infty} f\left(x\right)=l$ (l is a finite value)

Competency level 13.7 : Interprets infinite limits.

Number of periods : 01

Learning outcome : 1. Explains vertical asymptotes.

Guidelines to learning - teaching process :

1. When vertical asymptote occures :

   let $f(x)=\frac{p(x)}{q(x)}$.

   Let a be a zero of $q(x)$ and check whether

   $\lim_{x\to a^-} f\left(x\right)=\pm\infty$ and $\lim_{x\to a^+} f\left(x\right)=\pm\infty$

Note : If there are many zeros for $q(x)$, then there exist many vertical asymptotes for $f(x)$.

If there's no zero for $q(x)$,no vertical asymptote for $f(x)$.

Competency level 13.8 : Interprets continuity at a point.

Number of periods : 02

Learning outcome : 1. Explains continuity at a point by using examples.

Guidelines to learning - teaching process :

1. Explain that, if

   $$\lim_{x \to a} f(x) = \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$$

   Then the function is continous at $x = a$

Combined Mathematics II

Competency 2 : **Uses systems of coplanar forces.**

Competency level 2.4 : Explains the equilibrium of a particle under the action of three forces.

Number of periods : 05

Learning outcomes :
1. Explains equilibrium of a particle under the action of three coplaner forces.
2. States the conditions for equilibrium of a particle under the action of three forces.
3. States the triangle law of forces, for equilibrium of three coplanar forces.
4. States the converse of the triangle law of forces.
5. States Lami's theorem for equilibrium of three coplanar forces acting at a point.
6. Proves Lami's Theorem.
7. Solves problems involving equilibrium of three coplanar forces acting on a particle.

Guidelines to learning - teaching process :

1. State that a particle acted upon by a system of coplanar forces is in equilibrium, if the resultant of the system of forces is zero.

2. Show that the particle is in equilibrium under three coplanar forces only if the resultant of any two forces is equal in magnitude and opposite in direction to the third force.

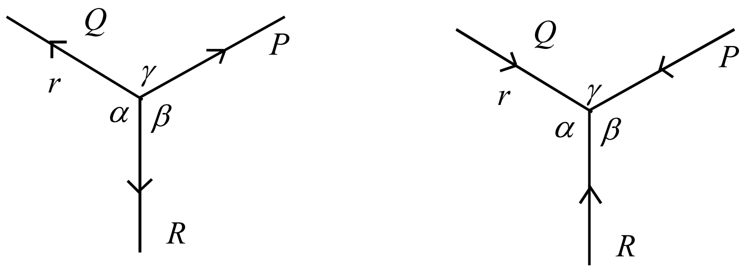
3. **Triangle law of forces**
   If three coplanar forces acting on a particle can be represented in magnitude and direction by the three sides of a triangle taken in order, then the three forces are in equilibrium.

   Proves the theorem of triangle law of forces.

4. **Converse of triangle law of forces**
   - If three coplanar forces acting on a particle are in equilibrium then they can be represented by three sides taken in order of a triangle in magnitude and direction.
   - Prove the converse of the theorem of triangle law of forces.
   - Direct students to solve problems using the theorem of triangle of forces and its converse.

5. **Lami's Theorem**

If three coplanar forces acting at a point are in equilibrium then magnitude of each force is directly proportional to the sine of the angle between the other two forces.



It the particle is in equilibrium under three forces P, Q, R then

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

6. Prove Lami's theorem.

7. Direct students to solve problems using
   (i) Theorem of triangle law of forces and its converse.
   (ii) Lami's theorem.

Competency level 2.5 : Explains the resultant of coplanar forces acting on a rigid body.

Number of periods : 04

Learning outcomes :

1. Describes a rigid body.
2. States the principle of transmission of forces.
3. Explains the translation and rotation of a force.
4. Defines the moment of a force about a point.
5. States the dimensions and units of moments.
6. Explains the physical meaning of moment.
7. Finds the magnitude of the moment about a point and its sense.
8. Represents the magnitude of the moment of a force about a point geometrically.
9. Determines the algebraic sum of the moments of the forces about a point in the plane of a coplanar system of forces.
10. Uses the general principle of moment of a system of forces.

Guidelines to learning - teaching process :

1. Describe a rigid body as one in which the distance between any two points remains unchanged when it is subjected to external forces of any magnitude.

2. Explain that a force acting on a rigid body can be considered as acting at any point on its line of action.
   - Also explain that the above phenomins is known as transmission of forces.

3. Show that a linear motion as well as rotation can be created by a force.

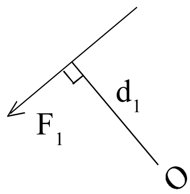
4. Present the definition that the moment of the force about a point is the product of the magnitude of the force and the perpendicular distance to its line of action from the given point.

5. Show that dimension is ML^2T^{-2} and the unit is Nm

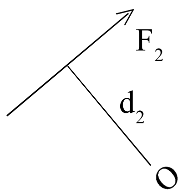
6. Build up the concept of moment as a measure of tendency to rotate about a certain point as a result of an external force acting on a rigid body. (For two dimensions only).

   Make them to understand that what is measured by moment is a turning effect about a line perpendicular to the plane determined by that point and the line of action of the force

7. Demonstrate that the sense of the moment can be considered as clockwise or anticlockwise. Explain that according to sign convention an anticlockwise moment is considered positive and a clockwise moment is considered negative.



Moment of F_1 about O $= F_1 \times d_1$.



Moment of F_2 about O $= F_2 \times d_2$

O $= -F_2 \times d_2$

8. Explain that the magnitude of the moment of a force F denoted in magnitude, direction and position by $\overrightarrow{AB}$ about a point O is twice the area of the triangle OAB.

9. Direct students to solve problems of finding the algebraic sum of the moments of a system of coplanar forces about a point in the plane.

10. State that the algebraic sum of moments of a system of coplanar forces about a point in the plane is equal to the moment of the resultant of the system about the same point. (Proof is not expected). Explain using suitable examples.

    - Guide students to solve problems involving moments

Competency level 2.6 : Explains the effect of two parallel coplanar forces acting on a rigid body.

Number of periods : 06

Learning outcomes :
1. Uses the resultant of two non parallel forces acting on a rigid body.
2. Uses the resultant of two parallel forces acting on a rigid body.
3. States the conditions for the equilibrium of two forces acting on a rigid body.
4. Describes a couple.
5. Describes the sense of a couple
6. Calculates magnitude and moment of a couple.
7. States that the moment of a couple is independent of the point about which the moment of the forces is taken.
8. States the conditions for two coplanar couples to be equivalent.
9. States the conditions for two coplanar couples to balance each other.
10. Combines coplanar couples

Guidelines to learning - teaching process :

1. When the two forces are **no parallel**
   As the two forces meet at a point show that their resultant can be determined by applying the parallelogram law of forces.

2. - When the two forces are **parallel**
     Introduce that forces acting on lines parallel to each other are known as parallel forces.

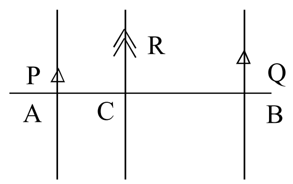
- When two parallel forces are acting in the same direction they are said to be like forces, while those acting in opposite directions are said to be unlike forces.

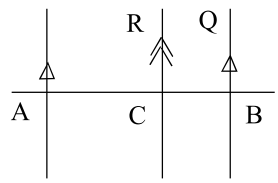
- Explain that the parallelogram law of forces cannot be used to find the resultant of two parallel forces.

- If the two forces are P and Q and their resultant is R, and if the lines of action of P, Q and R intersect a certain straight line at A, B and C respectively.

if P and Q are like, $R = P + Q$

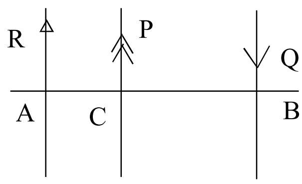
$P > Q$ $\qquad\qquad\qquad\qquad\qquad\qquad$ $P < Q$

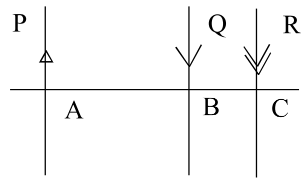




$P(AC) = Q(BC)$ $\qquad\qquad\qquad$ $P(AC) = Q(BC)$

- If *P* and *Q* are unlike
  if *(P>Q)*, $\qquad\qquad\qquad\qquad\qquad\qquad$ if *(P<Q)*





$R = P - Q$
$P.AC=Q(BC)$

$R = Q - P$
$P(AC)= Q(BC)$

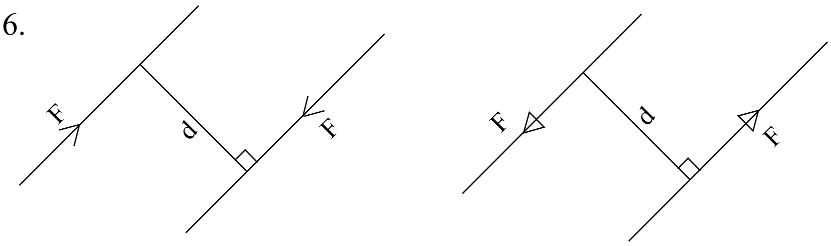
- Show that for two forces acting on a rigid body are in equilibrium, the two forces should be collinear, equal in magnitude and opposite in direction.

3. Introduce a couple as a pair of parallel unlike (opposite) forces of equal magnitude. In this case, show that the vector sum of the two forces is zero, the algebraic sum of the moments of the two forces about any point is not zero. Therefore there is no translation, but only a rotation of the body.

4. Show that the moment of a couple about any point in the plane of the couple is the product of the magnitude of one of the forces forming the couple and the perpendicular distance between the lines of action of the two forces.

5. Mention that according to sign convention the anticlockwise (left handed) moments are considered as positive whereas clockwise (right handed) moments are considered to be negative.

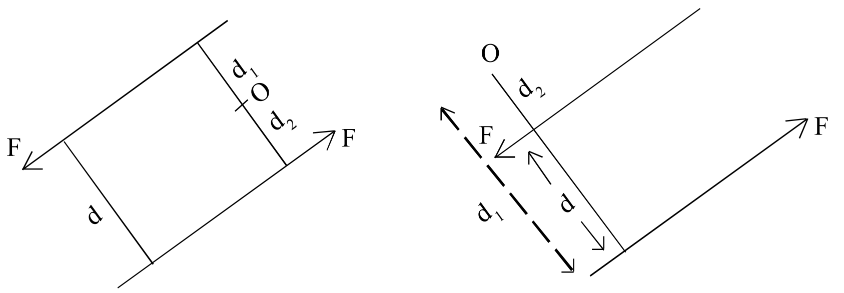
6.



Moment of the couple $= F \times d$ Moment of the couple $= F \times d$

$= -F \times d$ $= -F \times d$

7.



The moment of two forces about

$O = Fd_1 + Fd_2 = F(d_1 + d_2)$ $O = Fd_1 - Fd_2 = F(d_1 - d_2)$

$= F \times d$ $= F \times d$

Show that the same result will be obtained by taking moments about any point in the plane of the couple.

8. As two coplanar couples with moments of equal magnitude and same sense produce same rotation, they are equivalent.

9. As the resulting rotation when two coplanar couples of equal magnitude but of opposite sense, produce zero rotation, two such couples are said to balance each other.

10. Explain that the algebraic sum of moment should be taken in finding the moment of the combination of two or more coplanar couples.

Competency level 2.7 **:** Analysis of a system of coplanar forces acting on a rigid body.

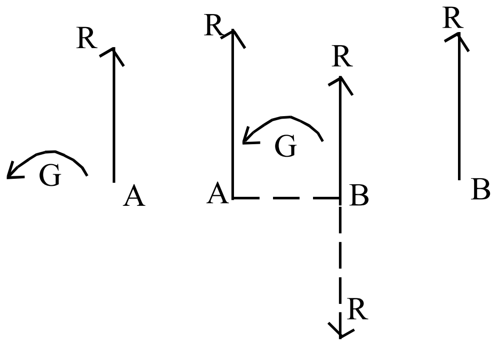
Number of periods **:** 08

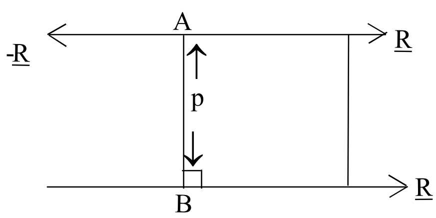
Learning outcomes **:**

1. Reduces a couple and a single force acting in its plane into a single force.
2. Shows that a force acting at a point is equivalent to the combination of an equal force acting at another point together with a couple.
3. Reduces a system of coplanar forces to a single force acting at O together with couple of moment G
4. Finds magnitude, direction and l ine of action of a system of coplaner forces.
5. Reduces a system of coplanar forces to a single force acting at a given point in that plane.
6. (i) Expresses reduction of a system of coplanar forces to a single force when it reduces to a single force.
   (ii) Expresses reduction of a system of forces to a couple.
   (iii) Expresses conditions necessary for equilibrium.
7. Solves problems involving rigid bodies under the action of coplaner forces.

Guidelines to learning - teaching process **:**

1.



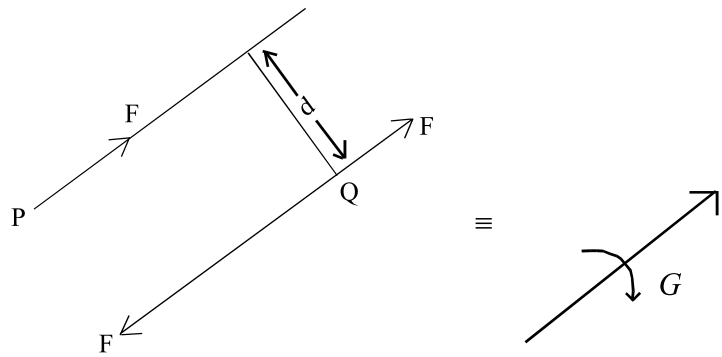


$$G = p \cdot \underline{R}$$

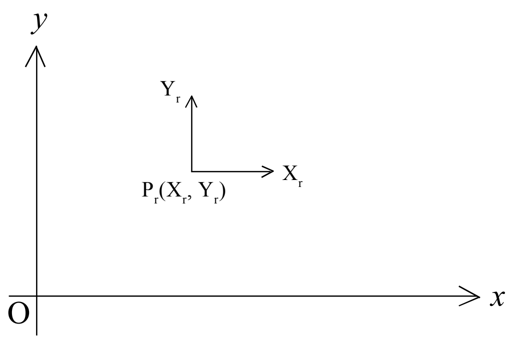
$$p = \frac{G}{R}$$

Show that a couple of moment G and a single force $\underline{R}$ acting in its plane is equivalent to a force equal and parallel to a force $\underline{R}$ acting at a distane $\frac{G}{R}$ from the point of action of it.

2. Show that the force F acting at P is equivalent to the combination of a force F acting at a point Q and a couple of moment $G = F \times d$, where d distance between the lines of action of the two forces F.



3. Show how a system of coplanar force is reduced to a single force R acting at origin O and a couple of moment G.



Consider the system of coplanar forces F, acting at $P_r(X_r, Y_r)$.
where r = 1, 2, ..., n

$$\underline{R} = \sum_{r=1}^{n} \underline{F}_r = \sum_{r=1}^{n} \left(X_r \underline{i} + Y_r \underline{j} \right)$$

$$= \left(\sum_{r=1}^{n} X_r \right) \underline{i} + \left(\sum_{r=1}^{n} Y_r \right) \underline{j}$$

$$= X \underline{i} + Y \underline{j}$$

Where $X = \sum_{r=1}^{n} X_r$ and

$$Y = \sum_{r=1}^{n} Y_r .$$

4. **Magnitude of R**

$$R = \sqrt{X^2 + Y^2}$$

R makes an angle θ with the X - axis

$$\theta = \tan^{-1}\frac{Y}{X}$$

$$G = \sum_{r=1}^{n}\left(x_r Y_r - y_r X_r\right) \text{ anti clockwise.}$$

Show that the equation of the line of action is

$$G - xY + yX = 0 .$$

5. Show how it can be reduced to a single force $R^{/}$ acting at the point P(x, y) in the plane and a couple $G^{/}$

6. Obtain that
$R^{/} = R$ and
$G^{/} = G - xY + yX$
Present the following necessary and sufficient conditions for two coplanar systems of forces to be equivalent.

That when each system of forces is resolved seperately along two axes Ox and Oy perpendicular to each other selected in the plane of the two systems of forces each algebraic sum of components of one system of forces is equal to the corresponding algebraic sum of the other system of forces.

$X^{/} = X, \ \ Y^{/} = Y$, where X and Y are the algebraic sums of one system of forces along Ox and Oy and $X^{/}$, $Y^{/}$ are the algebraic sums of the other system of forces along the same axes.

If the algebraic sum of the moments of the two systems of forces about any point (h, k) in the plane, are
$G_1^{/}$ and $G_2^{/}$, $G_1^{/} = G_2^{/}$

7. When a system of coplanar forces is reduced to a single force $\underline{R} = \left(X\underline{i} + Y\underline{j}\right)$ acting at the origin O and couple of moment G, discuss cases to reduce a system a of coplaner forces to a couple,

(i) $\underline{R} \neq \underline{0}$ (i.e. at least one of X or Y should not be zero). Show that when $G = 0$ it is reduced to a single force through origin and when $G \neq 0$ it is reduced to a single force through another point.

(ii) Show that $\underline{R} = \underline{0}$ (i.e., $X = 0$ and $Y = 0$) and $G \neq 0 \Rightarrow$ (system reduced to) reduces to a couple.

(iii) If $\underline{R} = 0$ (i.e., $X = 0$ and $Y = 0$) and $G = 0$.
Show that the systems of forces are in equilibruim.
Give suitable examples for each condition.

Competency 3 : Applies the Newtonian model to describe the instantaneous of motion in a plane

Competency level 3.1 : Uses graphs to solve problems involving motion in a straight line.

Number of periods : 08

Learning outcomes :

1. Defines "distance and speed"
2. States diamention and units of distance and speed.
3. Defines average speed.
4. Defines instantaneous speed.
5. Defines uniform speed.
6. States dimensions and standard units of speed.
7. States that distance and speed are scalar quantities.
8. Defines position coordinates of a particle undergoing rectilinear motion. (motion in a straight line)
9. Defines Displacement.
10. States the dimension and standard units of displacement.
11. Defines average velocity.
12. Defines instantaneous velocity.
13. Defines uniform velocity.
14. Expresses dimension and units of velocity.
15. Draws the displacement -time graph.
16. Draws velocity - time graph.
17. Finds the average velocity between two positions using the dis placement time graph.
18. Determines the instantaneous velocity (between two positions) using the displacement time graph.
19. Defines accelaration.
20. States the dimension and unit of acceleration.
21. Defines average acceleration.
22. Defines instantaneous acceleration.
23. Defines uniform acceleration.
24. Defines retardation.
25. Draws the velocity time graph.
26. Finds average accelaration using the velocity time graph.
27. Finds the acceleration at a given instant using velocity - time graph.
28. Finds displacement using velocity time graph.
29. Draws velocity time graphs for different types of motion.
30. Solves problems using displacement time and velocity-time graph.

Guidelines to learning - teaching process :

1. Distance: When a variable point P moves from a point A to a point B present the length measured along the path form A to B as the distance travelled by the point.

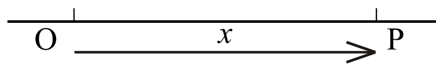
   Speed : Speed is defined as the rate of change of distance with respect to time.
   State dimensions and standard units of distance.

2. Dimensions of distance : L Standard unit is meter (m).
   Introduce and other units of distance such as mm, cm, km.

3. If the total distance travelled from A to B is *s,* and the time taken is *t* then define that $\frac{s}{t}$ as the average speed during the time interval *t.*

4. The speed of a moving particle at a given instant of time is defined as the instantaneous speed at that time.

5. If the instantaneous speed of a particle over a certain time interval remains constant, define that speed as a uniform speed.



6. Dimension of speed LT^{-1};
   Standard unit is ms^{-1}.
   Introduce other units of speed such as kmh^{-1}.

7. Distance is a quantity with a magnitude measured with some unit. Time is also same. Since they do not possess a direction demonstrate that distance and speed are scalar quantities.

8. The position coordinate of a point P moving along a line '*l*' is denoted by '*x*', and defined as $x = \pm OP$ depending whether its position is on the right or left of the point O.
   Explain that *x* is a function of time *t.*

9. Define displacement as the change in the position coordinate during a given time interval.

If the position coordinate of the particle P at times t_1 and t_2 are x_1 and x_2 respectively, show that the displacement s of the particle during the time interval $(t_1,\ t_2)$ is defined as $s=(x_2-x_1)$.
Explain that the displacement is a vector quantity.
Show also that accordingly as $s>0$ or $s<0$ its direction is taken as $\rightarrow or \leftarrow$.

10. Dimension L; Standard unit is m (meter)

Explain other units too in terms of the standard units (cm, km)

11. During the time interval $(t_1,\ t_2)$ let the displacement be $s=(x_2-x_1)$. Average velocity during that time interval is defined as $\frac{x_2-x_1}{t_2-t_1}$ and also show that average velocity can be positive or negative.
Average velocity too is a vector quantity.

The average velocity of a particle during a small interval $[t, t+h]$ is given by $\frac{x_{t+h}-x_t}{h}$

12. Average velocity of the particle when $h \rightarrow 0$

$$\lim_{n\rightarrow 0}\frac{x_{t+h}-x_t}{h}=V=\frac{ds}{dr}$$

13. Introduce this velocity as the instantaneous velocity of the particle at time 't'. Show that $v=\frac{ds}{dt}$ and explain that velocity is also a function of time.

If the displacement 's' is measured from a fixed point O then show that v can also be written as $v=\frac{ds}{dt}$,
velocity is the rate of change of displacement with respect to time.
By sign convention, accordingly as $v>0$ or $v<0$, its direction is taken as $\rightarrow or \leftarrow$.

14. Velocity: Velocity is defined as the rate of change of displace ment with respect to time.

15. Dimension of velocity v is LT^{-1} and standard unit is ms^{-1}. Also explain other units in terms of the standard unit (cms^{-1}, kmh^{-1})

16. Using suitable examples to explain that how displacement time graph is drawn.

17. If s_1 and s_2 are the displacements corresponding to the times t_1 and t_2 show that the average velocity $\frac{s_2 - s_1}{t_2 - t_1}$ can be obtained by the gradient of the line P_1P_2. Here P_1 and P_2 are points on the displacement - time curve corresponding to times t_1 and t_2 respectively.

18. Show that the gradient of the tangent drawn to the displacement time graph, at a particular point gives the instantaneous velocity at that moment.

    Obtain the relation instantaneous velocity $= \frac{ds}{dt}$

    $= \text{gradient}$

19. Acceleration: Define acceleration of the velocity - time curve at time t of the tangent as the rate of change of velocity with respect to time.
    Since velocity is a vector, acceleration also is a vector.

20. Express that the dimension of acceleration is LT^{-2} and the Standard unit is, meters per meter per second squared $\left(ms^{-2}\right)$. Mention other units of acceleration such as cms^{-2}, kmh^{-2}

21. If the velocities of a particle are v_1 and v_2 at times t_1 and t_2 respectively, then define the average acceleration of the particle during the time interval $[t_1, t_2]$ as $\frac{v_2 - v_1}{t_2 - t_1}$.

    Show that average acceration $\gtreqless 0$ accordingly as $V_2 \gtreqless V_1$

22. The average acceleration during a small time interval $\left[t, t+h\right]$ is

$$\frac{v_{t+h} - v_t}{h}.$$

As $h \to 0$, the limit of the average acceleration

$$\lim_{h \to 0} \frac{v_{t+h} - v_t}{h} = a$$

Introduce a as the instantaneous acceleration at time t.

Show that $a = \frac{dv}{dt}$. This is known as the acceleration of the particle at time t.

Explain that acceleration is a function of time.

Emphasize the fact that acceleration is the rate of change of velocity with respect to time.

23. If the acceleration is constant in a certain time interval, define that motion as a motion with uniform acceleration.

24. State that when the acceleration is negative it is called a retardation.

25. Provide an understanding of drawing velocity time graphs using suitable examples.

26. If the velocities corresponding to times t_1 and t_2 are v_1 and v_2 respectively, then show that the average acceleration during the time interval $\left[t_1,\ t_2\right]$ is $\frac{v_2 - v_1}{t_2 - t_1}$. It can be obtained as the gradient of the line P_1P_2. Here P_1 and P_2 are two points corresponding to times t_1 and t_2 in the velocity time curve.

27. Show that the gradient of the tangent drawn at a given point to the velocity time graph gives the instantaneous acceleration at the time represented by the point. Deduce that instantaneous acceleration

$a = \frac{dv}{dt}$ (gradient). Show also that $a = v\frac{dv}{ds}$.

28. Explain that displacement during a certain time - interval is given by the area between the graph and the time axis. (An area below the time axis is assigned a negative sign).

29. Direct students to draw velocity time graphs for
    1. Position of rest - zero velocity.
    2. Uniform velocity.
    3. Uniform acceleration.
    4. Uniform retardation.
    5. A combinations of such motions.

30. Direct students to solve problems related to rectilinear motion with uniform acceleration.

Competency level 3.2 : Uses kinematic equation to solve problems involving motion in a stright line with constant acceleration.

Number of periods : 08

Learning outcomes :
1. Derives kinematic equations for a particle moving with uniform acceleration.
2. Derives kinematic equations using velocity - time graph.
3. Uses kinematic equations for vertical motion under gravity.
4. Uses kinematic equations to solve problems.
5. Uses velocity - time and displacement - time graphs to solve problems.

Guidelines to learning - teaching process :

1. Introduce the standard symbols for
   initial velocity - u ; final velocity - v
   acceleration - a ; time - t and
   displacement - s and derive the kinamatic equations.

$$v = u + at$$

$$s = \frac{1}{2}(u + v)t$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2\ a\ s$$

2. Guide students to derive kinematic equations using velocity-time graph.

3. Show that in this case the acceleration should be replaced by g, (the acceleration due to gravity) which is a constant.
   Show that g is taken approximately as 10ms^{-2} and remind as sumptions.

4. Explain using suitable examples.

Competency level 3.3 : Investigates relative motion between bodies moving in a straight line with constant acceleration.

Number of periods : 06

Learning outcomes :

1. Describes the concept of frame of reference for two dimensional motion.
2. Describes the motion of one body relative to an other when two bodies are moving in a straight line.
3. States the principle of relative displacement for two bodies mov ing along straight line.
4. States the principle of relative velocity for two bodies moving along a straight line.
5. States the principle of relative acceleration for two bodies moving along a straight line.
6. Uses kinematic equations and graphs related to motion for two bodies moving along the same straight line with constant relative acceleration.

Guidelines to learning - teaching process :

1. Introduce that a particle moving (on a straight line) and an axis fixed rigidly to the particle so that it lies along the line constitute a frame of reference.

2. Explain with examples.

3. If the displacement of particles P and Q relative to the frame of reference O on a straight line are $s_{\text{P,O}}$ and $s_{\text{Q,O}}$ respectively express the displacement of Q relative to P as $s_{\text{Q,P}} = s_{\text{Q,O}} + s_{\text{O,P}}$.
   Show that $s_{\text{O,P}} = -s_{\text{P,O}}$.

4. Obtain $v_{Q,P} = v_{Q,O} + v_{O,P}$ by differentiating the displacement equation with respect to time.

5. Obtain the relation $a_{Q,P} = a_{Q,O} + a_{O,P}$ by differentiating the velocity equation with respect to time.
   Find relative displacement, relative velocity and relative aceleration for two particles moving along two parallel paths.

6. Explain using suitable examples.
   Consider only cases where the distance between the parallel paths is negligible.

7. Guide students to solves related problems.

Third Term

Combined Mathematics I

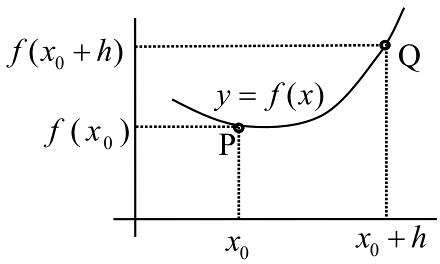
Competency 14 : Differentiates functions using suitable methods.

Competency level 14.1 : Describes the idea of derivative of a function.

Number of periods : 06

Learning outcomes :
1. Explains slope and tangent at a point.
2. Defines the derivative as a limit.
3. Explains rate of change.

Guidelines to learning- teaching process :



1. Let y be a function of x. Then x given by $y=f(x)$

   Let P be a point on the curve $y=f(x)$ with x coordinates equals to x_o So $P=\left(x_o, f(x_o)\right)$

   Let Q be a near by point on the curve $y=f(x)$

   Suppose that x coordinate of Q is $x+h$

   then $Q\equiv\left(x_0+h, f(x_0+h)\right)$

   Let m_{pq} denotes the slope of the secant line PQ,

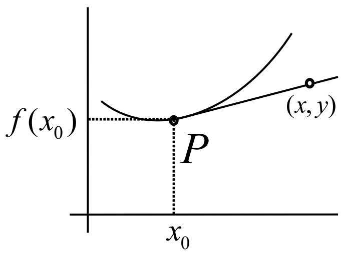
   $$\text{Then } m_{pq}=\frac{f(x_0+h)-f(x_0)}{x_0+h-x_0}\text{ ; for } h\neq 0$$

   $$=\frac{f(x_0+h)-f(x_0)}{h}\text{ ; for } h\neq 0$$

   Lim m_{pq}, when it is exist as a $h\to 0$ as a real number then it is defined to be the slope of m of the tangent line to the graph

   $y=f(x)$ at P

   $$m_{pq}=\lim_{h\to 0}\frac{f(x_0+h)\ -\ f(x_0)}{h}$$



equation of the tangent line at P to the curve $y = f(x)$ with slope m is, $y = f(x_0) \ = m(x - x_0)$

2. The limit $\lim\limits_{h \to 0} \dfrac{f(x_0 + h) - f(x_0)}{h}$ which was used to define the slope of the tangent line is given a name and a notation. Since it occurs in many other situations it is called derivative of *f(x),* at $x = a$, and it is denoted by $f(x_0)$ provided the limit exists (as a real number) $f(x)$ is differentiable at $x = x_0$.

   Using suitable examples , explain that the derivative of *f(x)* does not exist at $x = x_0$. in the instances given below.

   (i) When *f* is not defined in an open interval including $x = x_0$

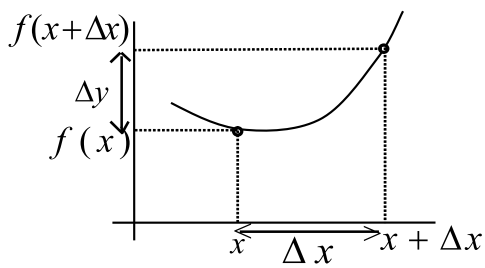
   (ii) When $\lim\limits_{h \to 0} \dfrac{f(x_0 + h) - f(x_0)}{h}$ is not finite

   (iii) When $\lim\limits_{h \to 0} \simeq \dfrac{f(x_0 + h) - f(x_0)}{h}$ is not defined

- The function f' whose domain consists of all values of *x* at which the derivative exists is called the derivative function of *f(x)* so $(f')(x) = f'(x)$ and

$$f'(x) = \lim_{h \to o} \frac{(x + h) \ - \ f(x)}{h}$$

3. Let *y* be a function of x given by $y = f(x)$. Take any x and consider an increment of Δx is consider the change in x in the closed interval with x and $x + \Delta x$ as its end points.



Δx is the change in x values the corresponding change in y values denoted by Δy is equal to $f(x+\Delta x)-f(x)$

Therefore the average rate of change in y with respect to x in the closed interval with x and $x+\Delta x$ as its end points is $\dfrac{\Delta y}{\Delta x}$

this $\dfrac{\Delta y}{\Delta x}=\dfrac{f(x+\Delta x)-f(x)}{\Delta x}$

Emphasize that Δx is a symbol and not a product of Δ and x
The Instantaneous rate of change of y with respect to x is defined to be $\lim\limits_{\Delta x\to 0}\dfrac{\Delta y}{\Delta x}=\lim\limits_{\Delta x\to 0}\dfrac{f(x+\Delta x)-f(x)}{\Delta x}$

provided the limit exist as real number

Note that $\lim\limits_{\Delta x\to 0}\dfrac{\Delta y}{\Delta x}=f'(x)$

$\lim\limits_{\Delta x\to 0}\dfrac{\Delta y}{\Delta x}$ is also denoted by $\dfrac{dy}{dx}$

Hence $f'(x)$ is the same as $\dfrac{dy}{dx}$

Competency level 14.2 : Determines the derivatives from the first principles.

Number of periods : 05

Learning outcomes : 1. Finds derivatives from the first principles.

Guidelines to learning - teaching process :

1. When n is a rational number, showing the method of determining the derivative of x^n and the derivatives of basic trigonometric functions from first principles. Present the proofs of the following results.

$$\frac{d}{dx}\left(x^n\right)=n\,x^{n-1} \qquad \frac{d}{dx}\left(\cot x\right)=-\operatorname{cosec}^2 x$$

$$\frac{d}{dx}\left(\sin x\right)=\cos x \qquad \frac{d}{dx}\left(\sec x\right)=\sec x\tan x$$

$$\frac{d}{dx}\left(\cos x\right)=-\sin x \qquad \frac{d}{dx}\left(\operatorname{cosec} x\right)=-\operatorname{cosec} x\cot x$$

$$\frac{d}{dx}\left(\tan x\right)=\sec^2 x$$

Competency level 14.3 : States and uses the theorems and rules of the differantiation.

Number of periods : 03

Learning outcomes :
1. States the basic rules of derivatives.
2. Solves problems using basic rules of derivatives.

Guidelines to learning - teaching process :

1. Prove that, When k is a constant
   - (i) When k is a constant and $f(x)=k$ then $f'(x)=0$.
   - (ii) If $f(x)=kg(x)$ then $f'(x)=kg'(x)$.
   - (iii) If $f(x)=g(x)\pm h(x)$ then $f'(x)=g'(x)\pm h'(x)$
   - Direct students to solve problems using $\dfrac{d(x^n)}{dx}=nx^{n-1}$ and the above theorems after explaining suitable examples.

2. (i) Product rule

$$\frac{d}{dx}[f(x)\cdot g(x)] = f(x)\frac{d}{dx}[g(x)]+g(x)\frac{d}{dx}[f(x)]$$

   (ii) Quotient rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)]-f(x)\frac{d}{dx}[g(x)]}{\{g(x)\}^2},$$

   Where $g(x)\neq 0$

   (iii) Chain rule

   If y is a function of u and u is a function of x then

$$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$$

   - Chain Rule and its extension proofs are not required of the above results.
   - Solves problems using above results.

Competency level 14.4 : Differentiates inverse trigonometric fucntions.

Number of periods : 03

Learning outcomes :
1. Finds the derivatives of inverse trigonometric functions.
2. Solves problems using the derivatives of inverse trigonomatric functions.

Guidelines to learning - teaching process :

1. Deduces that

   (i) $\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}, \quad -1 < x < 1$

   (ii) $\frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1-x^2}}, \quad -1 < x < 1$

   (iii) $\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}, \quad -\infty < x < \infty$

2. Differentiates various functions using the above formulae.

Competency Level 14.5 : Describes natural exponential function and find its derivative.

Number of periods : 02

Learning outcomes :
1. Defines the exponential function (e^x)
2. Express domain and range of the exponential function.
3. State that e is an irrational number.
4. Describes the properties of the e^x.
5. Writes the estimate of the value of e
6. Writes the derivative of the exponential function and uses it to solve problems.
7. Sketches the graph of e^{-x}

Guidelines to learning - teaching process :

1. States that the sum of the infinite series

   $1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots\dots + \frac{x^n}{n!} + \dots\dots$ is donated by e^x

   - e^x is called the natural exponential function.

2. State that the domain of the natural exponential function in $\mathbb{R}$ and the range is $(0, \infty)$

3. • by taking $x = 1$ we get

$$e = e^1 = 1 + \frac{1}{1!} + \frac{1}{2!} + ... + \frac{1}{n!} + ...$$

State that e is an irrational number and that $e \simeq 2.718$

4. State that
   (i) $e^0 = 1$
   (ii) $e^{x_1 + x_2} + e^{x_1} \ e^{x_2}$
   (iii) $e^{x_1 - x_2} = \dfrac{e^{x_1}}{e^{x_2}}$
   (iv) for rational values of r, $(e^x)^r = e^{rx}$
   (v) $\lim\limits_{x \to \infty} e^x = \infty$
   (vi) $\lim\limits_{x \to \infty} e^{-x} = 0$

5. $$f(1) = e = 1 + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + ... + \frac{1}{\lfloor n} + ... \simeq 2.718$$

   Stress that e is a positive irrational number.

6. States that $\dfrac{d}{dx} e^x = e^x$

   Solves problems involving natural exponential functions.

7. Guide students to draw the graph of $y = e^x$

   At this stage only the shape of the graph is needed

Competency level 14.6 : Describes natural logarithmic functions.

Number of periods : 03

Learning outcomes :
1. Defines the natural logarithmic functions.
2. Expresses the domain and range of the logarithmic functions.
3. Expresses the properties of $ln\, x$
4. The graph of $y = ln\, x$
5. Defines the function a^x for $a > 0$
6. Expresses the domain and range of the function a^x
7. Solves problems involving logarithmic functions.
8. Deduces the derviative of $ln\, x$
9. Deduces the derviative of a^x
10. Solves problems using the derviative of $ln\, x$ and a^x

Guidelines to learning - teaching process :

1. Explain that $ln\,x$ defined by $y = ln\;x \Leftrightarrow x = e^y$. $ln\;x$ is called the natural logarithimis function of e^x

2. If $g(x) = ln\;x$, then the domain of g is $(0, \infty)$ and the range is $\mathbb{R}$

3. (i) $ln\,x$ is edfined only for $x > 0$

   (ii) $ln\,(e^x) = x$, for $x \in \mathbb{R}$

   (iii) $e^{ln\,x} = x$ for $x > 0$

   (iv) $ln\,(x\,y) = ln\;x + ln\;y$ for $x > 0$ and $y > 0$

   (v) $ln\left(\dfrac{x}{y}\right) = ln\;x - ln\;y$ for $x > 0$ and $y > 0$

   (vi) $ln(x^b) = b ln\;x;\;\; x > 0$

4. Sketch the graph of $y = ln\;x$, using the inverse property.

   The graph of $y = ln\;x$, is the mirror image of $y = e^x$ on $y = x$

5. The function a^x is defined $a^x = e^{x \ln a}$

6. If $h(x) = a^x$, then the domain of $h = \mathbb{R}$ and the range of $h = (0, \infty)$

7. Solves problems involving natural logarithmic function.

8. Deduce that

   $$\frac{d(\ln x)}{dx} = \frac{1}{x},\; x > 0$$

9. Deduce that $\dfrac{d(a^x)}{dx} = (ln\;a)\,a^x$

10. Solves problems using the derivative of a^x

Competency level 14.7 : Differentiate implicits functions and parametric functions.

Number of periods : 06

Learning outcomes :

1. Define implicit functions.
2. Finds the derivatives of implicite functions.
3. Differentiates parametric functions.
4. Writes down the eqautions of the targent and normal at a given point to a curve.

Guidelines to learning - teaching process :

1. A function defined by $y = f(x)$, which statisfies an equation of the $F(x, y) = 0$ is called an impliciat function.

   Explain this using $x^2 + y^2 - 1 = 0$

2. To obtain the derivative of an implicit function $y = f(x)$ defined by an equation $f(x, y) = 0$, not necessary (sometimes not possible) it is to solve $f(x, y) = 0$, explicity for *y* in terms of *x* and then get the derivative. We instead differentiate the both sides of $f(x, y) = 0$ with the chain rule. Explain using examples

   A curve C is defined by the parametric equation $x = f(t)$ and $y = f(t)$ where *t* is a parameter.

   Then the derivative $\frac{dy}{dx}$ can be obtained from

   $$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} \text{ at points for which } \frac{dx}{dt} \neq 0.$$

   Show also that

   $$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} \text{ at points for which } \frac{dx}{dt} \neq 0.$$

   Explains using examples

3. Differentiation involving implicit functions and parametric forms for the parabola $y^2 = 4ax$, elipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the hyperabola

   $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $xy = c^2$

$y^2 = 4ax$	:	$x = at^2,\ y = 2at$
$\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$	:	$x = a\cos\theta,\ y = b\sin\theta$
$\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$	:	$x = a\sec\theta,\ y = b\tan\theta$
$xy = c^2$	:	$x = ct,\ y = \dfrac{c}{t}$

4. Obtains equations of tangent and normals at points on curves defined parametrically, including the above curves.

   Show how to sketch the above mentioned curves and explains their elementary properties.

Competency level 14.8 : Obtains derivatives of higher order.

Number of periods : 02

Learning outcomes :
1. Finds derivatives of higher order.
2. Differentiats functions of various types.
3. Finds relationship among various order of derivatives.

Guidelines to learning - teaching process :

1. If y is a function of x , then n^{th} order derivative of y is obtained by differentiating $y \cdot n$ times with respect to x. This is denoted by

   $$\frac{d^n y}{dx^n}\ \ or\ \ f^{(n)}(x)\ \ or\ \ y^{(n)}$$

2. Explain using examples

3. Guide students to solves problems involving higher order derivatives,

Competency 15 : Analysies the behaviour of a function using derivatives

Competency level 15.1 : Investigates the turning points using derivatives.

Number of periods : 05

Learning outcome :

1. Defines stationary points of a given function.
2. Describes increasing or decreasing function.
3. Describes local (relative) maximum and local minimum.
4. Employs the first derivative test to find the maximum and minimum points of a function.
5. States that there exists critical points, which are neither a local maximum nor a local minimum.
6. Introduces points of inflection.
7. Uses the second order derivative to test whether a turning point of a given function is a local maximum or a local minimum.

Guidelines to learning - teaching process :

1. A point at which the derivative of a function is zero is defined as a stationary point. So, $f(x)$ has a stationary point when $x = c$ provided that $f'(c) = 0$ explains with suitable examples.

2. Explain that a function $f(x)$ increasing on an interval I, if $f(x_1) \leq f(x_2)$ whenever $x_1, x_2, \in I$ with $x_1 < x_2$

   - Explain that it $f'(x) > 0$ for $x \in I$, then $f(x)$ is strictly increasing on I.

   - Explain that a function $f(x)$ is decreasing on an interval I it $f(x_1) \geq f(x_2)$ whenever $x_1, x_2, \in I$ with $x_1 < x_2$

   - Explain that it $f'(x) < 0$ for $x \in I$, then $f(x)$ is strictly decreasing on I.

3. - State that if there exists $\delta > 0$ such that $f(x) \leq f(c)$ for all $x \in (c-\delta, c+\delta)$, then $f(x)$ is said to have a maximum at $x=a$
   - State that if there exists $\delta > 0$ such that $f(x) \geq f(c)$ for all $x \in (c-\delta, c+\delta)$, then $f(x)$ is said to have a minimum at $x=a$

4. Describes the first derivative test for local maximum nor local minimum.

5. - States that there exists stationary points which are neighter local maximum nor local minimum.
   - Discuss with examples in which $f'(c)=0$ but the point which neither local maximum nor local minimum

6. Introduce the point of inflection.

7. - If $f'(a)=0$ *and* $f''(a)<0$ then there is a local maximum at $x=a$
   - If $f'(a)=0$ *and* $f''(a)>0$ then there is a local minimum at $x=a$
   - Guide to solves problems involving maxima and minima.

Competency level 15.2 : Investigates the concavity

Number of periods : 02

Learning outcomes : 1. Uses second derivative test to find concavity.

Guidelines to learning teaching process :

1. State that it $f''(x)>0$ for $x \in (a,b)$ then the graph of f is concave upwords, and if $f(x)<0$ for $x \in (a,b)$ then the graph of f is concave down words.

   Explain that a point of inflection is a point at which concavity changes

   Give examples to show that the derivative need not be zero at the point of intection.

Competency level 15.3 : Sketches curves.

Number of periods : 04

Learning outcome : 1. Sketches the graph of a function.

Guidelines to learning teaching process :

1. Direct students to draw graphs of function using the above principles.
   Examples involving horizontal and vertical asymptotes are also included.

Competency level 15.4 : Applies derivatives for practical situation

Number of periods : 04

Learning outcome : 1. Uses derivatives to solve real world problems.

Guidelines to learning teaching process :

1. Direct students to solve problems involving maximum and minimum in real world activities.

Combined Mathematics II

Competency 3 : Applies the Newtonian model to describe the instantaneous of motion in a plane

Competency level 3.7 : Interprets the motion of a projectile in a vertical plane.

Number of periods : 10

Learning outcomes :

1. Introduces projectiles.
2. Describes the terms "velocity of projection" and " angle of projection".
3. State that the motion of a projectile can be considered as two motions separately in the horizontal and vertical directions.
4. Applies the kinematic equations to interpret motion of a projectile.
5. Calculates the components of velacity of a projectile in a given time.
6. Finds the components of displacement of a projectile in a given time.
7. Calculates the maximum height of a projectile.
8. Calculates the time taken to reach the maximum height of a projectile.
9. Calculates the horizontal range of a projectile and its maximum.
10. States that in general there are two angles of projection for the same horizontal range for a given velocity of projection.
11. Finds the maximum horizontal range for a given speed.
12. For a given speed of projection finds the angle of projection giving the maximum horizontal range.
13. Derives Cartesian equations of the path of a projectile.
14. Finds the time of flight.
15. Finds the angles of projection to pass through a given point.

Guidelines to learning- teaching process :

1. Introduce a projectile as a particle or a body moving freely under gravity.

2. When a particle or a body is projected with a velocity u inclined at an angle α to the horizontal introduce that u is the velocity of projection and α is the angle of projection.

3. Explain that the velocity component is constant for the horizontal motion and the acceleration, (which is due to gravity), is con stant for the vertical motion. In fact, accelaration $a = g$ directed vertically down wards.

4. Show that the following equations can be used with $a = g$,

Horizontally $\quad s = ut \quad : \quad s = u\cos\alpha \cdot t$

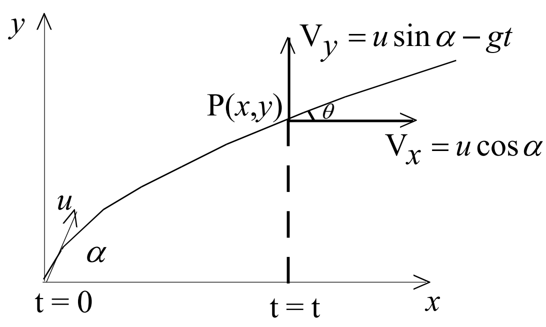
Vertically $\quad v = u + at \quad : \quad v = (u\sin\alpha)t - gt$

$$s = ut + \frac{1}{2}at^2 \ : \ y = (u\sin\alpha)t - \frac{1}{2}gt^2$$

$$v^2 = u^2 + 2a\,s : v^2 = u^2\sin^2\alpha - 2gy$$

s, u, v, a, t have usual meanings.

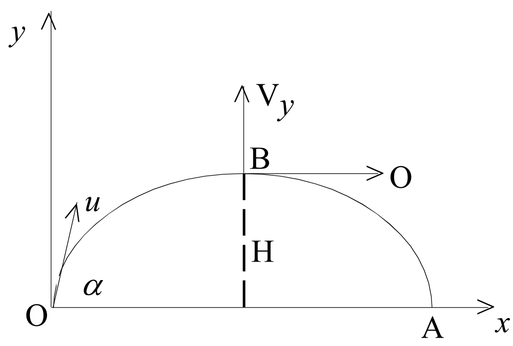
5.



Derive that the horizontal and vertical components of velocity at a time t, are $V_x = u\cos\alpha \cdot t$, $V_y = u\sin\alpha - gt$

6. Derive that the components of displacement at time t as x and y, state that these are parametric equations of the path of the projectile where "t" is the parameter.

7. Show that if H is the maximum height then $\mathrm{H} = \dfrac{u^2\sin^2\alpha}{2g}$



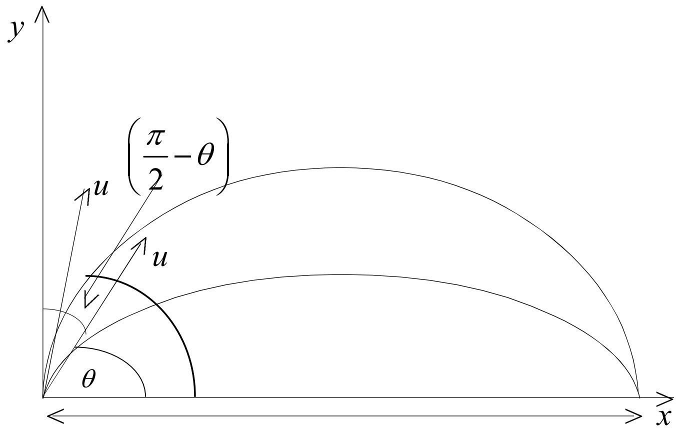
8. Show that if T is the time taken to reach the maximum height, then

$T_{0 \to B} = T$,

$T = \dfrac{u \sin \alpha}{g}$

9. If R is the horizontal range through the point of projection derive the expression for horizontal range R, $R = \dfrac{2u^2 \sin \alpha \cos \alpha}{g}$

10. Show that there are two angles of projection giving the same value for a given projection speed.



for a given u $\sin 2\theta = \left(\dfrac{gR}{u^2}\right)$

$\sin 2\theta = \sin 2\alpha$

$\therefore 2\theta = 2\alpha \text{ or } 180 - 2\alpha$

$\theta = \alpha \text{ or } 90 - \alpha$

$\sin 2\theta = \sin 2\alpha$

$2 \sin \theta \cos \theta = 2 \sin \alpha \cos \alpha$

$\therefore \theta = \alpha \text{ or } \theta = \dfrac{\pi}{2} - \alpha$

$\left[\begin{array}{c} \therefore \sin \theta = \sin \alpha \\ or \\ \sin \theta = \cos \alpha \end{array}\right]$

R which is obtained when $\alpha = \theta$ and $\alpha = \frac{\pi}{2} - \theta$ are substituted in the the expression for the same horizontal range

$$\mathrm{R} = \frac{2u^2}{g} \cos\alpha \sin\alpha$$

$$\therefore \text{ Time of flight } \frac{2u\sin\alpha}{g} = 2T$$

$$R = (u\cos\alpha)\frac{2u\sin\alpha}{g} = 2T$$

11. As $\mathrm{R} = \frac{u^2 \sin 2\alpha}{g} \leq \frac{u^2}{g}$ deduce that $\mathrm{R}_{\max} = \frac{u^2}{g}$, for given u.

12. Derive that the angle of projection giving the maximum horizontal range is $\frac{\pi}{4}$.

13. Derive the equation $y = x\tan\alpha - \frac{gx^2 \sec^2\alpha}{2u^2}$ by eliminating t from the equation obtained earlier for x and y

    when $\alpha \neq \frac{\pi}{2}$ compare this with the quadratic function

$y = ax - bx^2$, which represents a parabola through the point of projection.

Remind that $\alpha = \frac{\pi}{2}$ gives vertical motion under gravity.

14. If T' is the time taken to return to the level of the point of projection, then show that

$$\mathrm{T}' = \frac{2u\sin\alpha}{g} = 2\mathrm{T}.$$

15. Guide to find the angle of projection to passing through a point for a given velocity.

Competency 2 : **Uses systems of coplanar forces.**

Competency level 2.8 : Explains the equibirum of three coplanar forces acting on a rigid body as a particular case.

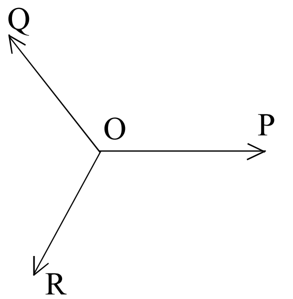
Number of periods : 08

Learning out comes :
1. States conditions for equilibrium of three coplanar forces acting on a rigid body.
2. Finds unknowns when a rigid body is in equilibrium using,
   - Triangle law of forces.
   - Lamis theorem, cotangent rule.
   - Geometrical properties.
   - Resolving in to directions perpendicular to each other.

Guidelines to learning - teaching process :

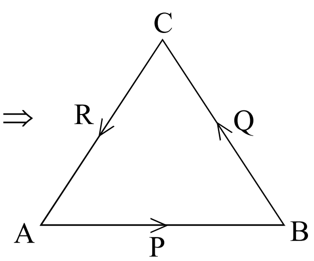
1. If three coplanar forces acting on a rigid body are in equilibrium then their lines of action passing through the same point or else they are parallel.
   Explain that this is a necessary condition only.

2. - State the triangle law of forces and its converse. (This has been presented under the equilibrium of a particle.)



Three forces P, Q, R acting at O are in equilibrium O

$\Rightarrow$



Corresponding triangle of forces, (ΔABC).

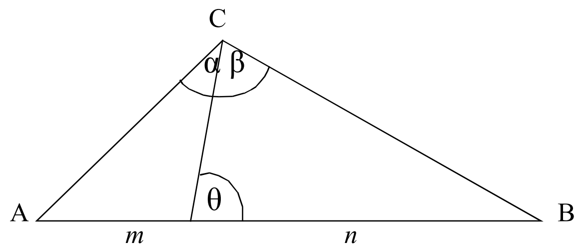
Using the triangle of forces, ABC, show that,

$$\frac{P}{AB} = \frac{Q}{BC} = \frac{R}{CA}$$

Use this result in solving problems.

- **Lami's** theorem (Presented under equilibrium of a particle)
  Explain that this theorem can also be applied in solving problems involving equilibrium of three coplanar non parallel forces whose lines of action are concurrent.

- **Cot rule**



Type 1: $n \cot A - m \cot B = (m+n)\cot\theta$

Type 2: $m \cot \alpha - n \cot \beta = (m+n)\cot\theta$

Show that, using examples the above results can be used in solving problems.

- In solving problems make use of the geometrical properties of the diagram as far as possible.

- Consider the equilibrium which is three forces are parallel. In that condition magnitude of resultant that of two forces is equal to the third force opposite in direction and their lines of action are same.

- State that the algebraic sums of resolutes along two perpendicular directions seperately are zero.

Competency level 2.9 : Investigates the effect of friction

Number of periods : 10

Learning out comes :

1. Describes "Friction" and "Frictional force".
2. Distinguishes between smooth and rough surfaces.
3. States the advantages and disadvantages of friction.
4. Whites the definion of "Limiting frictional force".
5. State laws of friction.
6. Defines angles of friction and coefficient of limiting friction.
7. Expresses the conditions for equilibrium.
8. Solves problems related to equlibrium of a particle or a rigid body, under the action of forces including frictional force.

Guidelines to learning - teaching process :

1. - Introduce frictional force as a force acting along the tan gential plane that prevents or tends to prevent relative motion between two bodies in contact. The property that produces this force is called friction.

   - State that as the applied force is gradually increased the frictional force too increases until the equilibrium ceases to exist

2. From the surfaces incontact, introduce that the surfaces without a frictional force as smooth surfaces and surfaces with a frictional force as rough surfaces.

3. Discuss the advantages and disadvantages of friction using suit able esamples.

4. Introduce "Limiting frictional force" as the masimum frictional force between two surfaces at the instant when relative motion between the two surfaces in contact is about to commence.

5. - When two bodies in contact under go motion relative to each other the frictional force exerted by one body on the other is opposite to the direction in which the body moves or tends to move relative to the other.

   - When in limiting equilibrium the frictional force is just suf ficient to prevent relative motion.

   - The ratio of the limiting frictional force to the normal reaction at the point of contact is a constant, called the coefficient of limiting friction and depends only on the material the surface of the bodies are made of.

   - As long as the normal reaction remains unchanged, the limiting frictional force does not depend on the area of contact or the shape of the two surfaces.

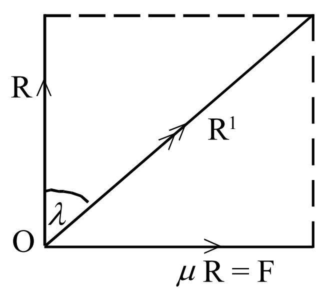
   - When the motion sets in the limiting frictional force decreases slightly.

   - When there is relative motion between the surfaces, the ratio of the frictional force and the normal reactions is slightly less than the coefficient of limiting friction.

6. • Define the coefficient of limiting friction between any two given surface as the ratio of the limiting frictional force to the normal reaction. If F_l is the limiting frictional force and R is the normal reaction then $\mu_l = \frac{F_l}{R}$ is called coefficient of limiting friction. This is also called the coefficient of static friction

• Introduce the angle of friction as the angle made by the resultant reaction with the normal reaction when the equilibrium is limiting.

• If the angle of friction is λ ; deduce the result $\mu_l = \tan \lambda$.



$$\tan \lambda = \frac{F}{R}$$
$$= \mu$$

7. • States the conditions for equlirium

• Express that the frictional force F between two surfaces in contact at any time is given by $F \leq \mu_l$. (equality occuring at limiting equilibrium) and $F \leq \mu_l R$.

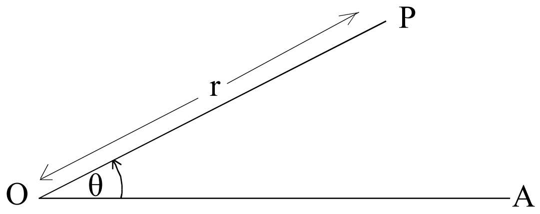
8. Direct students to solve problems involving friction.

Competency level 3.4 : Explain the motion of partcle on a plane

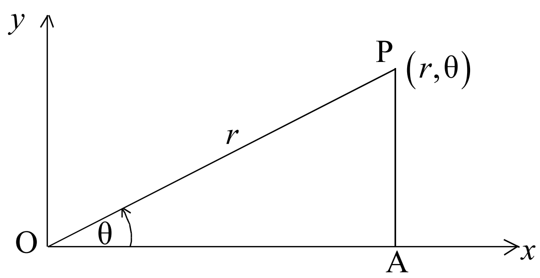
Number of periods : 06

Learning out comes :
1. Finds relation between the Cartesion coordinates and the polar coordinates of a particle moving on a plane
2. Finds the velocity and accelaration when the position vector is given as a function at time.

1.



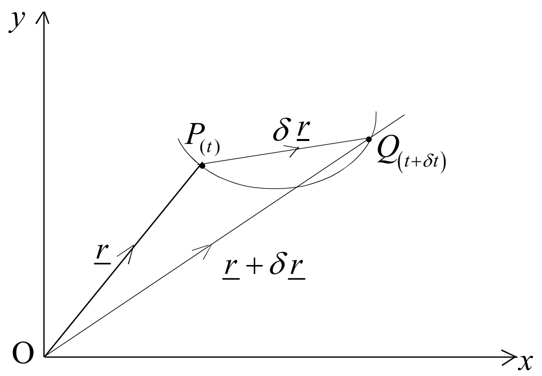
- If A is a fixed point, OA is a fixed line and P is a variable point such that $OP = r$ and $\measuredangle AOB = \theta$ then state that the polar coordinates of the point P is defined as (r, θ).
  Where $r \geq O$, and the angle θ is measured anti clock wise is taken to be positive. Show that a point can also be denoted uniquely by polar coordinates.



- with respect to the system of axes OXY if $P \equiv (x, y)$ then derive that $x = r\cos\theta$, $y = r\sin\theta$ and also derive $\underline{r} = r\left(\cos\theta\underline{i} + \sin\theta\underline{j}\right)$

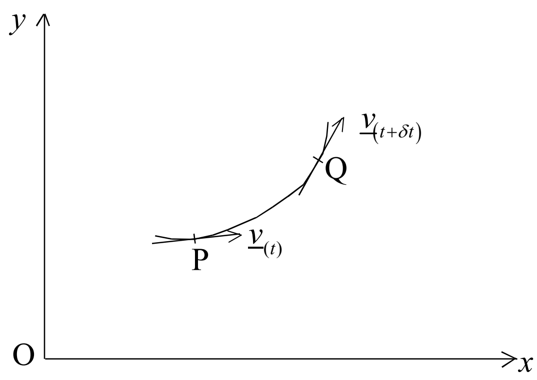
2. - Consider a system of coordinate axes OXY in the plane of the motion of particle P. Take unit vactors along OX and OY as $\underline{i}$ and $\underline{j}$ respectively. Then (x, y) are the position coorfinates of P then the position vector of P is given by $\underline{r} = x\underline{i} + y\underline{j}$ states that x and y are functions on time. then $\underline{r} = x(t)\underline{i} + y(t)\underline{j}$.

- Take the position of a point at time t, relative to origin O is P and its position at time $t+\delta t$ as Q Here, if $\overrightarrow{OP} = \underline{r}$,Show that the average velocity in the time interval can be expressed as $= \dfrac{\overrightarrow{PQ}}{\delta t} = \dfrac{\delta \hat{\underline{r}}}{\delta t}$.



4. States that the onstantaneous velocity of the particle at time t is detined as

$$\underline{v} = \lim_{\delta t \to 0} \frac{(\underline{r}+\delta \underline{r}) - \underline{r}}{\delta t} = \lim_{\delta t \to 0} \frac{\delta \underline{r}}{\delta t} = \frac{d\underline{r}}{dt}$$



- States that the postion of the particle at time *t* and $t+\delta t$ are P and Q respectively and corresponding velocities are $\underline{v}_t$, $\underline{v}_{(t+\delta t)}$.

- Define the average accelaration of the partucale in the time interval It is equal to $\dfrac{\underline{v}_{(t+\delta t)} - \underline{v}_{(t)}}{\delta t}$

- The instantaeous accelartion of a particle at time t is define as

$$\underline{a} = \lim_{\delta t \to 0} \frac{\underline{v}_{(t+\delta t)} - \underline{v}_{(t)}}{\delta t} = \lim_{\delta t \to 0} \frac{\delta \underline{v}}{\delta t} = \frac{d\underline{v}}{dt}$$

- If the velocity $\underline{v}_{(t)}$ at a time t is represented by $\overrightarrow{\text{LM}}$ and the velocity $v_{t+\delta t}$ at time $t+\delta t$ by $\overrightarrow{\text{LN}}$ (in magnitude and direction) then MN reprsents $\underline{v}_{t+st} - v_t$ Emphasize that as $\lim_{\delta t \to 0} \frac{\underline{v}_{(t+\delta t)} - \underline{v}_{(t)}}{\delta t}$ gives the acceleration. MN gives the direction of the accelaration as $\delta t \to 0$. ie the acceleration of the particle is directed towards the concave side of its path.

Competency level 3.5 : Determines the relative motion of two particles moving on a plane.

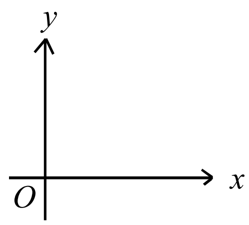
Number of periods : 10

Learning out comee :

1. Defines frame of referance
2. Obtains the displacement ,velocity and accelaration relative to a frame of refrence
3. Explains the principles of relative displacement, relative velocity and relative accelaration
4. Finds the path and velocity of a particle relative to another particle

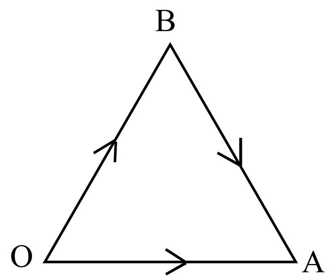
Guidelines to learning - teaching process :

1. Consider a body A moving in a plane. Select Cartesian axes perpendicular to each other in the plane of motion and rigidly fixed to the body A. Introduce that a fixed set of points relative to the axes (can be extended at will) is know as the frame of reference of A.



2. - Remind the earlier definitions of displacement, velocity and acceleration. Explain that if the displacement of a particle relative to the origin of a frame of reference is $\underline{r}$ than the $\underline{v} = \frac{d\underline{r}}{dt}$ and the acceleration $\underline{a} = \frac{d\underline{v}}{dt}$.
   - Explain using suitable examples.

3. • Show that if $\underline{r}_{A,O}$ and $\underline{r}_{B,O}$ are the position vectors of A and B respectively, relative to the origin O then the position vector of B relative to A, $\underline{r}_{B,A}$ is given by $\underline{r}_{B,A} = \underline{r}_{B,O} + \underline{r}_{O,A}$



- Derive the velocity of B relative to A, $\underline{v}_{B,A} = \underline{v}_{B,O} - \underline{v}_{O,A}$
- By differentiating the velocity equation with respect to time, de rive that $\underline{a}_{B,A} = \underline{a}_{B,O} - \underline{a}_{O,A}$.
- Direct students to find the velocity of one body relative to an other body when the relative velocity is uniform.

4. Direct students to find the path of one body relative to another body when the relative velocity is uniform.

Competency level 3.6 : Uses Principles of relative motion to solve real world problems.

Number of periods : 10

Learning out comes :
1. Finds the vector equation of a line on which a body is moving.
2. Finds the shortest distance between two particles .
3. Finds the requirement for collision of two particles
4. uses vectors to solves problems involving realative velocity

Guidelines to learning - teaching process :

1. • Direct students to find the following when the relative velocity is uniform.
   • Velocity of a particle relative to another or body.

2. • Shortest distance between two particles and time taken to reach it .

3.
- The time taken to meat (If it so) and their position when they met.
- Time taken to conplete a given path.

4.
- Present solutions of problems involving motion relative to water or air.
- Guide students to solves problems using vectors.

Competency level 3.8 : Applies Newton's laws to explain motion relative to an inertial frame

Number of periods : 10

Learning outcomes :
1. States Newton's first law of motion.
2. Defines "force".
3. Defines "mass".
4. Defines linear momentum of a particle.
5. States that linear momentum is a vector quantity.
6. States the dimensions and unit of linear momentum.
7. Describes an inertial frame of reference.
8. States Newton's second law of motion.
9. Defines Newton as the absolute unit of force.
10. Derives the equation $\underline{F} = m\underline{a}$ from second law of motion.
11. Explains the vector nature of the equation $\underline{F} = m\underline{a}$ from second law of motion.
12. States the gravitational units of force.
13. Explains the difference between mass and weight of a body.
14. Describes "action" and "reaction".
15. States Newton's third law of motion.
16. Solves problems using $\underline{F} = m\underline{a}$
17. Bodies in contact and particles connected by light invetensible strings
18. Solves problems involving pullys
19. Solves problems involving wedges

Guidelines to learning - teaching process :

1. A body continues to be in a state of rest, or of uniform motion along a straight line, unless it is a acted upon by an external force.

2. Define 'force' as the external action that changes the state of motion of a body.

3. Define mass as the amount of response of the body to a force acting on it.

4. Define linear momentum of a particle of mass *m* moving with a velocity *u* as *mu*.

5. Show that in the product *mu*, the velocity $\underline{u}$ is a vector and hence the linear momentum is also a vector.

6. The dimensions of linear momentum are $\left[\mathrm{MLT}^{-1}\right]$.
Unit of linear momentum is $kgms^{-1}$.

7. Describe an inertial frame of reference as a frame, which is at rest or moving with a uniform velocity relative to a frame of refer ence fixed on the earth.

   [Earth is considered as an inertial frame of reference in studying motions taking place on Earth's - surface.]

8. Tha rate of change of linear momentum of a body is directly proportional to the external force. Derive $\underline{\mathrm{F}} = m\underline{a}$ from the second law.

9. The Newton is defined as the force required to produce an ac celeration of 1 ms^{-2}, on a mass of 1 kg. Introduce this as an absolute unit of force.

10. From Newton's second law of $\mathrm{F} = k\ ma$ and according to the definition of the Newton, show that $k = 1$. Therefore $\mathrm{F} = ma$.

    Hence *m* is measured in kilograms *a* in meters per second $\left(ms^{-2}\right)$ then F is measured in Newtons.

11. According to $\underline{\mathrm{F}} = \mathrm{m}\underline{a}$ the acceleration takes place in the direc tion of the force $\underline{\mathrm{F}}$. Show that the equation could be used by resolving the force and the acceleration in any direction.

12. Introduce *kg* weight as the gravitational force with which a mass of 1*kg* is pulled towards the earth, gravitational attraction.

13. Introduce that the mass of a body is the amount of matter contained in it and that the weight of a body is the graviational force with which that body is attracted towards the earth. State that when the mass is measured in kg, and the acceleration due to gravity is measured in ms^{-2} the units of weight is obtained in N.

14. Draw the attention of students to different types of forces - using examples show how forces always occur in pairs. For example, action and reaction between two bodies in contact.

15. "For every action between bodies, there is a reaction of equal magnitude and opposite direction along the same line of action"

16. Solves problems using $f = ma$

17. Students are expected to solve problems related to following cases.
    - i. Find acceleration of a body under the action of given external forces. Find external force when the acceleration is given . Find reaction between body and a lift, when the lift is moving with given acceleration. The acceleration pro duced by external forces acting on two bodies connected by a string and the tension in the string.
    - ii. When forces act on a system of bodies, find the accelera tion of each body of the system and reactions between bodies in the system.
    - iii. Calculate frictional forces acting, by considering the motion of a body on a rough plane.
    - iv. Problems relating to the motion of systems comprising of connected particles which have various accelerations or rigid bodies.
    - v. Motion of particle on a smooth wedge, which may (includ ing smooth pulleys) moving on a smooth plane.
    - vi. External forces acting on two bodies which are connected by a strings, obtain their acceleration and the tension.

18. Solves problems involving pullys

19. Solves problems involving wedges