G. C. E. (Advanced Level)

Combined Mathematics

Grade 13

Teacher's Instructional Manual

(To be implemented from 2010)

Department of Mathematics
Faculty of Science and Technology
National Institute of Education
Sri Lanka

COMBINED MATHEMATICS

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Content

First Term

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
		Inequalities including Modulii	
11.3	1 States the modulus (absolute value) of a real number.	Let $x \in \mathbb{R}$ Define $\lvert x\rvert = x, \quad \text{if } x \geq 0$ $\qquad = -x, \text{ if } x < 0$	06
	2 Defines the modulus functions.	Let $f : \mathbb{R} \to \mathbb{R}$ be a function $\lvert f\rvert$ is defined as follows. $\lvert f\rvert : \mathbb{R} \to \mathbb{R}$ $\lvert f\rvert(x) = \lvert f(x)\rvert$ $\lvert f\rvert(x) = f(x), \text{ if } f(x) \geq 0$ $\qquad = -f(x), \text{ if } f(x) < 0$ Illustrate with examples.	
	3. Draws the graphs of modulus functions.	Graphs of the functions such as $y = \lvert ax\rvert, \quad y = \lvert ax+b\rvert, \quad y = \lvert ax\rvert + b$ $y = \lvert ax+b\rvert + c$ $y = c - \lvert ax+b\rvert$ $y = \lvert ax+b\rvert + \lvert cx+d\rvert$ $y = \lvert ax^2+bx+c\rvert$ where $a, b, c, d \in \mathbb{R}$.	
	4. Solves inequalities involving modulus.	Determination of solution set of inequalities such as $\lvert ax+b\rvert \geq \lvert cx+d\rvert$ $\lvert ax+b\rvert \geq lx+m$ $\lvert ax+b\rvert \pm \lvert cx+d\rvert \geq k$ (i) algebraically (ii) graphically where $a, b, c, d \in \mathbb{R}$.	
		Straight Line	
27.1	1. Interprets the gradient (slope) of a line.	Define the gradient m of a line joining two points (x_1, y_1) and (x_2, y_2) to be $\dfrac{y_2 - y_1}{x_2 - x_1}$ provided that $x_1 \neq x_2$. Explain that if θ is the angle between a straight line and the positive direction of x axis, then $m = \tan\theta$	05

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
	2. Derives the various forms of equation of straight line.	• Straight line with gradient m and intercept c at y axis is $y = mx + c$ • Straight line with gradient m and passing through the point (x_1, y_1) is $y - y_1 = m(x - x_1)$ • Straight line passing through two points (x_1, y_1) and (x_2, y_2) is $y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$ provided that $x_1 \neq x_2$. If $x_1 = x_2$ then it is $x = x_1$. • Straight line with intercept on x and y axes are a and b respectively is $bx + ay = ab$. • The perpendicular form of a straight line $x\cos\alpha + y\sin\alpha = p$, p is the length of the perpendicular from the origin and α is the angle which that perpendicular makes with the positive direction of x axis. • General form $ax + by + c = 0$.	
27.2	1. Finds the coordinates of point of intersection of two lines.	Solve the linear simultaneous equations to find the coordinates of point of intersection of the corresponding straight lines.	02
	2. Derives the equation and applies to problems.	Derive that the equation of a straight line passing through the point of intersection of two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is $\lambda(a_1x + b_1y + c_1) + \mu(a_2x + b_2y + c_2) = 0$ where λ, μ are parameters.	
27.3	1. Identifies the positions of two points with respect to a given straight line.	Given a straight line $ax + by + c = 0$ and two points (x_1, y_1) and (x_2, y_2) show that the points lie on the same sides or opposite sides of the given line accordingly $(ax_1 + by_1 + c)(ax_2 + by_2 + c) \gtrless 0$.	02

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
27.4	1. Defines the angle between two straight lines.	State that there are two angles between two intersecting lines. Generally one is acute and the other is obtuse.	02
	2. Obtains a formula to find the angle between two straight lines.	Derive the angle between two straight lines $y = m_1x + c_1$ and $y = m_2x + c_2$ is $\tan^{-1}\left\|\dfrac{m_1 - m_2}{1 + m_1m_2}\right\|$, provided $m_1m_2 \neq -1$. Two lines with slopes m_1 and m_2 are (i) parallel if and only if $m_1 = m_2$ (ii) perpendicular if and only if $m_1m_2 = -1$.	
27.5	1. Writes the parametric equation of a straight line.	Show that the parametric equation of a straight line is $x = x_1 + r\cos\theta, y = y_1 + r\sin\theta$. where θ is the angle the line makes with the positive direction of the *x* axis and $\mathrm{AP} = \|r\|$. For $ax + by + c = 0$, $\dfrac{y - y_1}{a} = \dfrac{-(x - x_1)}{b} = t$ Where *t* is a parameter.	10
	2. Finds the perpendicular distance from a point to a straight line.	Show that the perpendicular distance of a point (h,k) from a line $ax + by + c = 0$ is $\dfrac{\|ah + bk + c\|}{\sqrt{a^2 + b^2}}$. Deduce that the distance beween two parallel lines $ax + by + c = 0$ and $ax + by + d = 0$ is $\dfrac{\|c - d\|}{\sqrt{a^2 + b^2}}$.	
	3. Derives the image of a point on a straight line.	Show that the image of a point (α, β) on the line $lx + my + n = 0$ is $(\alpha + lt,\ \beta + mt)$ where $t = \dfrac{-2(l\alpha + m\beta + n)}{l^2 + m^2}$.	

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	4. Obtains the equations of bisectors of the angle between two intersecting lines.	Show that the equations of bisectors of the angles between two intersecting lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ are $\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}=\pm\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$.	
		Circle	
28.1	1. Defines circle as a locus.	Define a circle as the locus of a point which moves on a plane such that its distance from a fixed point is always a constant. Fixed point is the centre of the circle and the constant distance is the radius of the circle.	02
	2. Obtains the equation of a circle.	Equation of the circle with centre (a,b) and radius r is $(x-a)^2+(y-b)^2=r^2$ If centre is the origin, the equation becomes $x^2+y^2=r^2$.	
	3. Interprets the general equation of a circle.	General equation of a circle is $x^2+y^2+2gx+2fy+c=0$. Show that the centre is $(-g,-f)$ and radius $\sqrt{g^2+f^2-c}$, where $g^2+f^2-c\geq 0$.	
	4. Finds the equation of the circle when the end points of a diameter is given.	Show that the equation of the circle with the points (x_1,y_1), (x_2,y_2) as the ends of a diameter is $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$.	
28.2	Identifies the position of a point with respect to a circle.	Given a point $P=(x_0,y_0)$ and the circle $S\equiv x^2+y^2+2gx+2fy+c=0$, explain that the point P lies inside the circle or on the circle or outside the circle accordingly as $x_0^2+y_0^2+2gx_0+2fy_0+c \lesseqgtr 0$	01
28.3	1. Discusses the position of a straight line with respect to a circle.	Let $U=lx+my+n=0$ be the straight line and $S\equiv x^2+y^2+2gx+2fy+c=0$ be the circle.	03

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		By considering, (i) discriminant of the quadratic equation in x or y, obtained by solving S = 0 and U = 0. (ii) radius of the circle and the distance between the centre of the circle and the straight line. Discuss whether, (a) the line intersects the circle (b) the line touches the circle (c) the line lies outside the circle; in both situations (i) and (ii).	
	2. Obtains the equation of the tangent at a point on a circle.	Show that the equation of the tangent at $\mathrm{P} = (x_0, y_0)$ on $\mathrm{S} \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ is $xx_0 + yy_0 + g(x + x_0) + f(y + y_0) + c = 0.$	
28.4	1. Finds the length of the tangent drawn to a circle from an external point.	Let $\mathrm{S} \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ and $\mathrm{P}(x_0, y_0)$ be an external point. Show that the length of the tangent is $\sqrt{x_0^2 + y_0^2 + 2gx_0 + 2fy_0 + c}$.	04
	2. Finds the equation of the tangents drawn to a circle from an external point.	Obtain the equations of tangents drawn to a circle from an external point.	
	3. Obtains the equation of the chord of contact of the tangent.	Let $\mathrm{S} \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ $\mathrm{P} = (x_0, y_0)$ Show that the equation of chord of contact of tangents drawn from P is $xx_0 + yy_0 + g(x + x_0) + f(y + y_0) + c = 0.$	
28.5	Interprets the equation $\mathrm{S} + \lambda u = 0$.	Explain that $\mathrm{S} + \lambda u = 0$ represents a circle that passing through the points of intersection of the circle S = 0 and the straight line $u = 0$. where λ is a parameter.	03

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28.6	1. States the conditions to decide the position of two circles.	Let C_1 and C_2 be centres of two circles with radii r_1 and r_2 respectively. (i) If the circles touch externally, then $C_1C_2 = r_1 + r_2$ (ii) If they touch internally, then $C_1C_2 = \|r_1 - r_2\|$ (iii) If they intersect, then $\|r_1 - r_2\| < C_1C_2 < r_1 + r_2$ (iv) If one lies within the other $C_1C_2 < \|r_1 - r_2\|$ (v) If each lies outside the other $C_1C_2 > r_1 + r_2$	10
	2. Obtains the condition for two circles to intersect orthogonally.	Define the angle between two intersecting circles. Show that if two circles $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ and $S' \equiv x^2 + y^2 + 2g'x + 2f'y + c' = 0$ intersect orthogonally, then $2gg' + 2ff' = c + c'$	
	3. Finds the equations of common tangents.	Show that the common tangent at the point of contact of the circles S = 0 and $S' = 0$ is $S - S' = 0$ Derive the equations of common tangents of two circles.	
28.7	Interprets the equation $S + \lambda S' = 0$	Let the equations of circles be $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ and $S' \equiv x^2 + y^2 + 2g'x + 2f'y + c' = 0$ (a) If they intersect and $\lambda \neq -1$ then $S + \lambda S' = 0$ represents circles passing through the points of intersection of S = 0 and $S' = 0$, where λ is a parameter. If they intersect and $\lambda = -1$ then $S + \lambda S' = 0$ represents the common chord.	02

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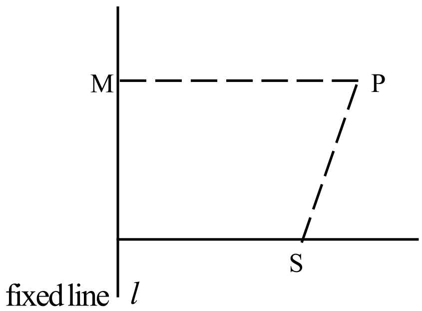
		(b) If they touch and $\lambda \neq -1$, then $S+\lambda S' = 0$ represents circles passing through the point of contact of the two circles. If they touch and $\lambda = -1$ $S+\lambda S' = 0$ represents the common tangent at the point of contact of the two circles, where λ is a parameter.	
29.0	1. Defines conic as the locus of a point.	**Conics** Definition: The set of points in a plane whose distances from a fixed point bear a constant ratio to the corresponding perpendicular distances from fixed straight line is a conic.  Let S be a fixed point and l a fixed straight line. From any point P, the perpendicular PM is drawn to the line l. The locus of P such that $\frac{SP}{PM}$ = constant, by definition is a conic. Fixed point is called the focus. Fixed line is called the directrix Constant ratio is called the eccentricity (e) If $e = 1$ conic section is a parabola If $0 < e < 1$ conic section is an ellipse If $e > 1$ conic section is a hyperbola	03
	2. Obtains the equations of conic section.	Derive the equations, $y^2 = 4ax$ is parabola $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is ellipse, where $b^2 = a^2\left(1-e^2\right)$	

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		$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ hyperbola, where $b^2=a^2\left(e^2-1\right)$ Discuss the coordinates and directrix of each conic section. Assymptotes of hyperbola. When $b=a$, the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ becomes $x^2-y^2=a^2$. The standard form of rectangular hyperbola is $xy=c^2$, where *c* is a parameter.	

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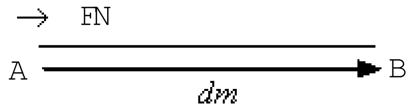
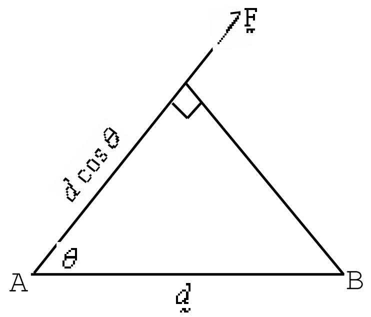
		Work, Power, Energy	
3.10	1 Explains the concept of work.	Explain the idea of work that the point of application moves under the action of a force is doing work.	08
	2 Defines work done under a constant force and its units.	Work is defined as the product of the constant force and the distance through which the point of application moves in the direction of the force.  Work done = Fd Nm The unit of force is *newton* and the unit of distance is *metre*. So that the unit of work done by a force is *newton metre*. This unit is called a *Joule* (J). Dimensions are ML^2T^{-2}.  Work done = $Fd\cos\theta$ J = $\underset{\sim}{F}.\underset{\sim}{d}$	
	3 Explains the Energy.	The energy of a body is its capacity for doing work. The SI unit of energy is the *Joule*. 1 kJ = 1000 J Note that both work and energy are scalar quantities. Work and energy are interchangeable and so the unit and dimension are same as work.	

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	4 Explains the Mechanical Energy.	Explain that we deal with mechanical energy only (except heat, light, sound and electrical energy) and mechanical energy is of two types; Kinetic Energy (K.E.) Potential Energy (P.E.)	
	5 Defines Kinetic Energy.	Kinetic is the capacity of a body to do work by virtue of its motion. It is measured by the amount of work that the body does in coming to rest. Obtain the formula K.E. = $\frac{1}{2}mv^2$, where m is the Mass and v is the Velocity. Explain that work done = change in Kinetic Energy.	
	6 Defines the Potential Energy.	The Potential Energy (P.E) of a body is the energy it possesses by virtue of its position. It is measured by the amount of work that the body would do in moving from its actual position to some standard position.	
	7 Explains the Gravitational Potential Energy.	Define the Gravitational potential energy as when a body of mass m is raised through a vertical distance h it does an amount of work equal to mgh.	
	8 Explains the Elastic Potential Energy.	Elastic Potential Energy is a property of stretched strings and springs or compressed springs. The amount of Elastic Potential Energy (E.P.E.) stored in a string of natural length a and modulus of elasticity λ when it is extended by a length x is equivalent to the amount of work necessary to produce the extension. Obtain that EPE $= \frac{1}{2}\lambda\frac{x^2}{a}$. EPE is always positive whether due to extension or to compression	
	9 Explains conservative forces.	Certain forces have the property that the work done by the forces is independent of the path (For an example weight) such forces are termed as conservative forces.	

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	10. Explains conservation of Mechanical Energy and applies to solve problems.	The principle of conservation of the mechanical energy for a system of bodies in motion under the action of a conservative system of forces, the sum of the kinetic energy and the potential energy of the system remains constant. K.E. + P.E. = Constant Application of the principle of conservation of mechanical energy.	
3.11	1. Defines Power and its units.	Define that the power is the rate of doing work. The power is measured in *Joule per second* (JS^{-1}) and this is called a *Watt* (W). Dimensions are ML^2T^{-3}.	07
	2. Explains the tractive force.	The tractive force as the producing force from the vehicle engine (driving force).	
	3. Derives the formula for power.	Relationship between power, driving force and velocity. If a force F N moves a body with a V ms^{-1} in the direction of the force then P = Fv (Unit of P in watts) Guide the students to solve problems in work, Power and Energy.	
		Impulse	
3.12	1. Explains the Impulsive action.	Define that impulse of a Constant Force as the product of the force and time, Δt. $\underset{\sim}{I} = \underset{\sim}{F}\Delta t$ Hence, obtain $\underset{\sim}{I} = m(\underset{\sim}{v} - \underset{\sim}{u})$ where m is the mass of the particle. $\underset{\sim}{I} = \underset{\sim}{F}\Delta t = \Delta(m\underset{\sim}{v})$	08
	2. States the units and Dimension of Impulse.	Units of impulse is Ns Dimension is MLT^{-1}. As Impulse is a vector when applying the formula $\underset{\sim}{I} = \Delta(m\underset{\sim}{v})$ the directions of forces and the velocities must be taken in consideration.	
	3. Defines the Principle of conservation of linear-momentum.	If vector sum of external forces is equal to zero or if there are no external forces acting on a system of bodies in a particular direction, the	

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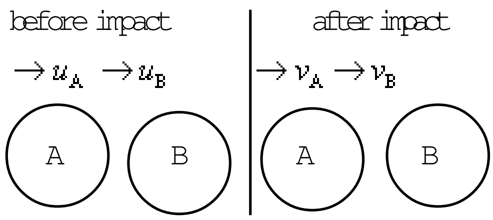
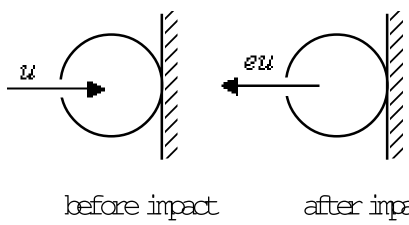
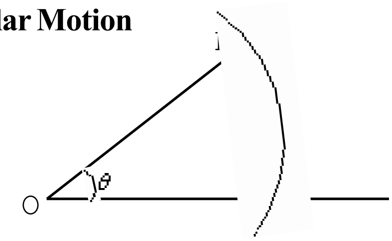
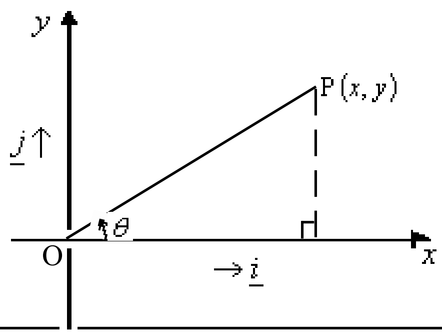
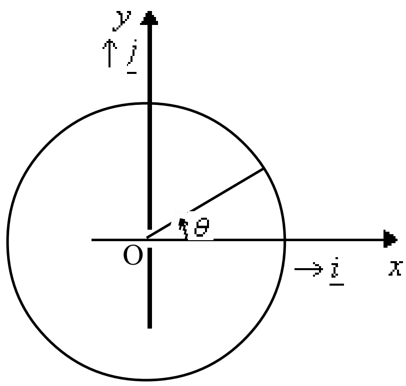
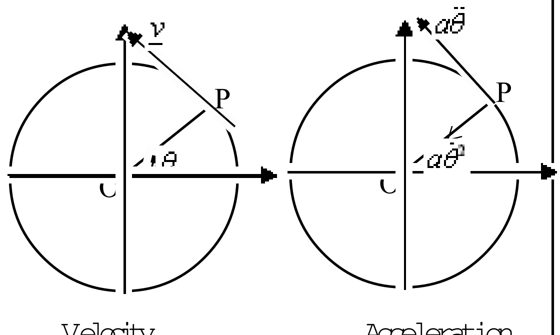
		total momentum of the system in that direction remains constant.	
	4 Finds the change in Kinetic energy due to impulse.	State that the change in K.E. is equal to $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$ $\Delta E = \frac{1}{2}m\left(v^2 - u^2\right) = \frac{1}{2}\underline{I}.\left(\underline{v} + \underline{u}\right)$ Solve problems on Impulse.	
		Direct Impact	
3.13	1. Explains direct impact.	Direct impact occurs when the directions of the velocities of the spheres just before the impact are along the line of centres on impact.	15
	2. States Newton's law of restitution.	When two bodies impinge directly, the relative velocity of separation after the impact bears a constant ratio to relative velocity of approach before the impact.	
	3. Defines coefficient of restitution.	The constant ratio is called coefficient of restitution and denoted by e.  $v_B - v_A = e\left(u_A - u_B\right)$ The constant e depends only on the material of which the bodies are made. $0 \leq e \leq 1$ If $e = 1$ the bodies are said to be perfectly elastic. If $e = 0$ the bodies are said to be in-elastic.	
	4. Explains the direct impact of a sphere on a fixed plane.	 	

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
		The velocity after impact is equal to e (velocity before impact) and in the opposite direction.	
	5 Calculates the change in kinetic energy.	During direct impact between two bodies of masses m_1 and m_2 the loss of kinetic energy due to impact is $\Delta E = \frac{1}{2}\frac{m_1 m_2}{(m_1+m_2)}(1-e^2)v^2$, where v is the relative velocity at the time of impact. If $e=1$, then $\Delta E = 0$	
3.14	1. Defines the angular velocity and acceleration of a particle moving in a plane.	**Circular Motion**  Let O be a fixed point and OA is a fixed line. If a particle moves in this plane then the angular velocity of P about O is defined to be the rate at which the angle POA increases is denoted by $\omega = \frac{d\theta}{dt} = \dot{\theta}$. Its units are given by (rad/s) Angular acceleration is defined as the rate of increase of angular velocity. Angular acceleration given by $\frac{d}{d\theta}\left(\frac{d\theta}{dt}\right) = \frac{d^2\theta}{dt^2}$ $\frac{d\dot{\theta}}{d\theta} = \ddot{\theta}$ Its units are given by rad/s^2	10
	2. Writes the relation between polar coordinates and cartesian coordinates and manipulates vector operations.	 	

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
	3. Finds the velocity and acceleration of a particle moving in a circle.	Let $P \equiv (a, \theta)$ $P \equiv (a\cos\theta, a\sin\theta)$ $\overrightarrow{OP} = a\cos\theta\underset{\sim}{i} + a\sin\theta\underset{\sim}{j}$ $= a\left[\cos\theta\underset{\sim}{i} + \sin\theta\underset{\sim}{j}\right]$ Define the unit vecot l in the direction of $\overrightarrow{OP}$. $\underline{l} = \cos\theta\underset{\sim}{i} + \sin\theta\underset{\sim}{j}$ Show that $\frac{d\underline{l}}{dt} = \dot{\theta}\underline{m}$ and $\lvert m\rvert = 1$, $\underline{m}$ is perpendicular to l.  P moves in a circle and OP = a (constant) $\underline{r} = a\left[\cos\theta\underset{\sim}{i} + \sin\theta\underset{\sim}{j}\right]$ $= a\underline{l}$ Show that the velocity $\underline{v}$, $\frac{dr}{dt} = \underline{v} = a\dot{\theta}\underline{m}$ and acceleration $\underline{f} = -a\dot{\theta}^2\underline{l} + a\ddot{\theta}\underline{m}$ and interpret the result.  Velocity Acceleration $\underline{v} = a\dot{\theta}$ along the tangent.	

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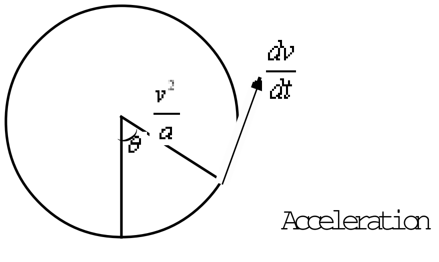
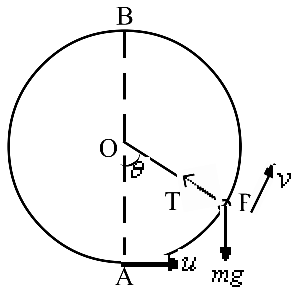
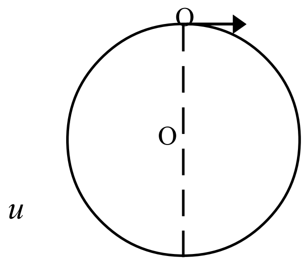
		Acceleration: 1. Component towards the centre is $a\dot{\theta}^2$ and 2. along the tangent is $a\ddot{\theta}$	
	4. States the velocity and acceleration of a particle moving with uniform speed in a circle.	Explain that the velocity $a\dot{\theta}$ is along the tangent and the speed is uniform. $a\dot{\theta}$ is constant. It follows that $\dot{\theta}$ is constant. Hence $\ddot{\theta}$ is zero. Velocity $V = a\dot{\theta} = a\omega$ Acceleration $a\dot{\theta}^2 = a\omega^2 = \frac{v^2}{a}$ towards the centre.	
	5. Finds the magnitude and direction of the force on a particle moving in a horizontal circle with uniform speed.	Explain that since the particle moves with uniform speed, the acceleration is towards the centre and a force must be acting towards the centre and this force is called centrifugal force.	
	6. Solves problems involving motion in a horizontal circle.	Guide students to solve problems involving the motion in a horizontal circle including conical pendulum.	
3.15	1. Explains vertical motion.	When a particle moves in a vertical circle of radius a, with varying velocity v, the acceleration towards the centre of the circle is $\frac{v^2}{a}$ and $\frac{dv}{dt}$ in the direction of tangent. $\left[\frac{v^2}{a} = a\dot{\theta}^2,\ \frac{dv}{dt} = a\ddot{\theta}\right]$  	10

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
	2. Explains the motion of a ring threaded on a fixed smooth vertical wire / particle moving in a fixed smooth circular, vertical tube.	Motion restricted to circular path. Explain that the only external force acting on the particle is Reaction. As the reaction is perpendicular to the direction of motion, it does no work. 1. Law of conservation of energy can be applied. 2. Applying $\underline{F} = m\underline{a}$ in the radial direction R can be found. Since the ring cannot leave the wire, the only condition necessary for it to describe complete circle is that its velocity is greater than zero at the highest point. Let u be the velocity at lowest point. (a) If $u^2 > 4ag$ it describes a complete circle. (b) If $u^2 < 4ag$ come to instantaneous rest before reaching highest point and subsequently oscillates.	
	3. Finds the condition for the motion of a particle suspended from an inelastic light string attached to a fixed point, in vertical circle.	 Let u be the velocity of particle m in the horizontal direction at the lowest point. When the string has turned through an angle θ, let v be the velocity and T be the tension. Using conservation of energy and applying $\underline{F} = m\underline{a}$ in the radial direction, obtain $v^2 = u^2 - 2ag(1-\cos\theta)$ $T = \frac{m}{a}\left[u^2 - 2ag + 3ag\cos\theta\right]$ Discuss the following. 1. If $u^2 \le 2ag$, the string is always taut, particle oscillates below the level of O.	

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
	                4 Discusses the motion of a particle on the outersurface of a fixed smooth sphere in a vertical great circle.	2 If $2ag < u^2 < 5ag$, then t becomes zero before v becomes zero and hence the string becomes slack. When the string is slack $\frac{\pi}{2} < \theta < \pi$. Once the string is slack, the only force acting on the particle is its own weight and motion continues as that of a projectile. If $u^2 \geq 5ag$ then the particle moves in a complete circle. Note that motion of a particle in a vertical circle on the innersurface of a smooth sphere is also same as above.  Let O be the centre of sphere and a be the radius. A particle is projected with velocity u in horizontal direction from the highest point of a smooth sphere. Discuss the motion and show that (i) If $u^2 \geq ag$, then the particle leaves the sphere at the point of projection (highest point) (ii) If $u^2 < ag$, then the particle leaves the sphere, when the radius through the particle makes an angle α with upward vertical, where $\alpha = \cos^{-1}\left(\frac{u^2 + 2ag}{3ag}\right)$. Solve problems leading to vertical circular motion.	

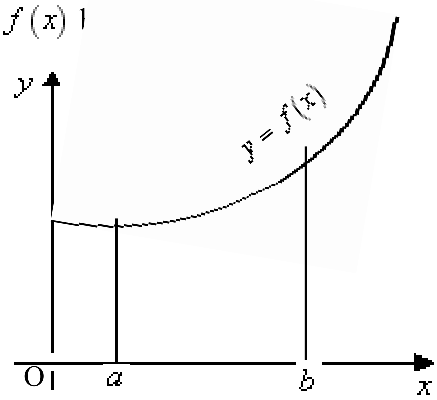
Second Term

COMBINED MATHS I (2nd Term)

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
25.1	1 Defines integration the reverse process of differentiation.	**Integration** If $\frac{d}{dx}\left[F(x)\right]=f(x)$, then $F(x)$ is an antiderivative of $f(x)$.	03
	2. Explains that any two antiderivatives of a function on an interval can differ by a constant.	The antiderivative of a function is not unique. It can differ by a constant.	
	3. Defines the indefinite integral as the collection of all antiderivatives.	If $\frac{d}{dx}\left[F(x)\right]=f(x)$, then we write $\int f(x)dx = F(x)+C$, where c is an arbitrary constant.	
	4. Identifies the standard forms of indefinite integrals.	1.(a) $\int x^n dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$ (b) $\int \frac{1}{x}dx = \ln\lvert x\rvert + C\ (x \neq 0)$ (c) $\int e^x dx = e^x + C$ 2.(a) $\int \sin x\, dx = -\cos x + C$ (b) $\int \cos x\, dx = \sin x + C$ (c) $\int \sec^2 x\, dx = \tan x + C$ (d) $\int \operatorname{cosec}^2 x\, dx = -\cot x + C$ (e) $\int \sec x \tan x\, dx = \sec x + C$ (f) $\int \cot x \operatorname{cosec} x\, dx = -\operatorname{cosec} x + C$ 3.(a) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C,$ $(-a < x < a)$ (b) $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$ $(a \neq 0)$	

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
	5. Explains the integrals when $px+q$ stands for x in standard forms of indefinite integrals.	If $\int f(x)dx = g(x) + C$ then $\int f(px+q)dx = \frac{1}{p}g(px+q)+C$ where $p \neq 0$	
25.2	States and explains basic rules of integration.	If f and g are functions of x, and k is a constant then 1. $\int k.f(x)dx = k\int f(x)dx$ 2. $\int[f(x)+g(x)]dx = \int f(x)dx + \int g(x)dx$	03
25.3	1. Defines the definite integral (using second version of the fundamental theorem of calculus) and states its uses to evaluate definite integrals.	If $\phi(x)$ is an antiderivative of $f(x)$ then $\int_a^b f(x)dx = [\phi(x)]_a^b = \phi(b)-\phi(a)$	02
	2. States the basic properties of the definite integrals.	(i) $\int_a^b f(x)dx = -\int_b^a f(x)dx$ (ii) $\int_a^b kf(x)dx = k\int_a^b f(x)dx$ (iii) $\int_a^b \{f(x)+g(x)dx\} = \int_a^b f(x)dx + \int_a^b g(x)dx$ (iv) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ when $a<c<b$ (v) $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ (Proof of (v) is required)	
25.4	1. Integrates rational functions when the numerator is the derivative of the denominator.	$\int \frac{f'(x)}{f(x)}dx = \ln\lvert f(x)\rvert + c$ where $f'(x)$ is the derivative of $f(x)$.	05
	2. Integrates rational functions using partial fractions.	$\int \frac{P(x)}{Q(x)}dx$ where $Q(x)$ is a polynomial of degree ≤ 4 and factorisable.	

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
25.5	Integrates trigonometric functions.	Using trigonometric identities to obtain the standard integrals. $\int \tan x\, dx,\ \int \cot x\, dx,\ \int \sec x\, dx$ $\int \operatorname{cosec} x\, dx,\ \int \sin^2 x\, dx,\ \int \cos^2 x\, dx$ $\int \tan^2 x\, dx,\ \int \cot^2 x\, dx$ $\int \sin^3 x\, dx,\ \int \cos^3 x\, dx,\ \int \sin mx \cos nx\, dx$ $\int \cos mx\ \cos nx\, dx,\ \int \sin mx\ \sin nx\, dx$	03
25.6	Integrates by substitution.	Use suitable substitution. (i) $\int \sin^m x\, dx$ (m is an odd positive integer) substitution $t = \cos x$ (ii) $\int \cos^m x\, dx$ (m is odd) substitution $t = \sin x$ (iii) $\int \sin^m x\ \cos^n x\, dx$ Where m, n are positive integers. If m is odd put $t = \cos x$ If n is odd put $t = \sin x$ (iv) $\int \frac{dx}{a \cos x + b \sin x + c}$ substitution $t = \tan \frac{x}{2}$ $\int \frac{dx}{a \cos^2 x + b \sin^2 x + c}$ substitution $t = \tan x$ (v) $\int \sqrt{a^2 - x^2}\ dx$ substitution $x = a \sin \theta$ or $a \cos \theta$ (vi) $\int \frac{dx}{\sqrt{a^2 + x^2}}$ substitution $x = a \tan \theta$ (vii) $\int \frac{1}{\sqrt{x^2 - a^2}}$ substitution $x = a \sec \theta$	04

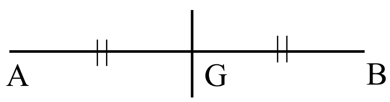
Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
		(viii) $\int \frac{dx}{(px+q)\sqrt{ax+b}}$ substitution $t = \sqrt{ax+b}$ (ix) $\int \frac{dx}{(px+q)\sqrt{ax^2+bx+c}}$ substitution $px+q = \frac{1}{t}$ and other substitutions	
25.7	Integrates by using the method of integration by parts.	If $u(x)$ and $v(x)$ are differentiable then show that $\int u\left(\frac{dv}{dx}\right)dx = uv - \int v\left(\frac{du}{dx}\right)dx$ Problems using integration by parts.	05
25.8	1. Finds the area under a curve.	Defines the area under a curve as a definite integral. Let $y = f(x)$  The area of the region bounded by the curve $y = f(x)$, the x axis and the lines $x = a$ and $x = b$ is $\int_a^b f(x)dx$ This is referred to as the area under the curve $y = f(x)$ from $x = a$ to $x = b$.	
	2. Finds the area between two curves.	Suppose $y = f(x)$ and $y = g(x)$ are two curves such that $f(x) \geq g(x)$ in the interval $[a,b]$. The area bounded by the two curves and the lines $x = a$ and $x = b$ is $\int_a^b \{f(x) - g(x)\}dx$.	

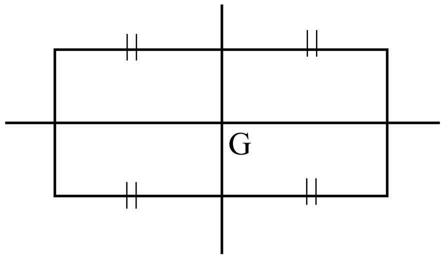
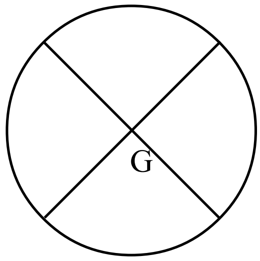
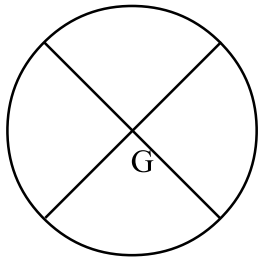
Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
		Permutation and Combination	
8.1	1. Defines factorial.	Definition of factorial n Normal form : $0! = 1$ $n! = 1.2.\ 3...n$ Recursive form : $F(0) = 1$ $F(n) = nF(n-1)$ where n is a positive integer.	02
	2. Explains the fundamental principle of counting.	Fundamental principle of counting: If one operation can be performed in m different ways and a second operation can be performed in n different ways, then there will be $m \times n$ different ways performing the two operations in succession.	
8.2	1. Defines ${}^nP_n$ and obtain the formulae for ${}^nP_n$.	Define that the number of permutations of n different objects taken all at a time is ${}^nP_n$ and ${}^nP_n = n!$.	06
	2. Defines ${}^nP_r$ and finds formulae for ${}^nP_r$.	Define that the number of permutations of n different objects taken r $(0 \le r \le n)$ at a time is ${}^nP_r$ and show that ${}^nP_r = \dfrac{n!}{(n-r)!}$.	
	3. Finds the permutations in which the quantities may be repeated.	Show that the number of permutations of n different objects taken r at a time when each object may occur any number of time is n^r .	
	4. Finds the permutations of n objects not all different.	Show that the number of permutations of n objects p of which are one kind and the remaining all are different is $\dfrac{n!}{p!}$.	
	5. Explains the circular permutations.	Show that the number of permutations in which n different objects can be arranged round a circle is $(n-1)!$	
8.3	1. Defines combination.	Define that the number of combinations of n different objects taken r at a time is ${}^nC_r$ and show that ${}^nC_r = \dfrac{n!}{(n-r)!\ r!}$	07

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
		$^nP_r = r!\,^nC_r$ Show that (i) $^nC_r = {}^nC_{n-r}$ (ii) $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$	
	2. Explains the distinction between permutations and combinations.	Explain (with examples) that in permutation, the order is important, but in combination order is immaterial. Show that the total number of combinations of n different objects taken any number at a time is $2^n - 1$. Guide students to solve problems on permutations and combinations.	
		Series	
21.1	1. Defines a sequence.	Definition of a sequence as a set of terms in a specific order with rule for obtaining terms. If a_n is the n^{th} term of a sequence, the sequence is denoted by $\{a_n\}$. $\{a_n\}$ is said to be convergent, if $\lim_{n \to \infty} a_n$ exists (finite number). Otherwise the sequence is said to be divergent.	04
	2. Defines an infinite series using the sequence of partial sum.	Connection between a sequence and a series. Let $\{a_n\}$ be a sequence. Define $S_n = \sum_{r=1}^{n} a_r \qquad n = 1, 2, 3, \ldots$ This is called the n^{th} partial sum.	
	3. Finds the sum of an arithmetic series.	Definition of an arithmetic series. A series, which after the first term, the difference between each term and the preceding is constant is called an arithmetic series or arithmetic progression. Show the general term T_r, $T_r = a + (r-1)d$, where a is the first term and d is the common difference and the sum of n terms	

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
		$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ $= \frac{n}{2}[a+l]$ where l is the last term of the series.	
	4. Finds the sum of a geometric series.	Definition of a geometric series. A series, which after the first term, the ratio between each term and the preceeding term is constant is called geometric series. (i) Show that the general term $T_p = ar^{p-1}$ where a is the first term and r is the common ratio. (ii) Show that the sum of n terms S_n, $S_n = \frac{a(1-r^n)}{(1-r)} (r \neq 1)$ $= na \ (r = 1)$	
21.2	Finds the sum of arithmetico-geometric series.	Give examples to arithmetico geometric series and discuss how to find the sum of n terms of an arithmetico geometric series.	02
21.3	1. States fundamental theorems on summation.	Show that (i) $\sum_{r=1}^{n}(u_r + v_r) = \sum_{r=1}^{n} u_r + \sum_{r=1}^{n} v_r$ (ii) $\sum_{r=1}^{n} ku_r = k\sum_{r=1}^{n} u_r$ where k is a constant. In general, $\sum_{r=1}^{n}(u_r v_r) \neq \sum_{r=1}^{n} u_r \sum_{r=1}^{n} v_r$	03
	2. Finds the sum of the series.	Determination of $\sum_{r=1}^{n} r, \ \sum_{r=1}^{n} r^2, \sum_{r=1}^{n} r^3$ and the use of above results and theorem [Examples (i) $\sum_{r=1}^{n} r(2r+3)$ (ii) $\sum_{r=1}^{n} 2r(r+1)(r+2)$]	

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
21.4	1. Uses various methods to find the sum of series.	Find summation of series using (i) method of difference (ii) partial fractions (iii) mathematical induction	10
	2. Discusses the sum of terms to infinity.	Let $\sum_{n=1}^{\infty} u_n$ be a series and $S_n = \sum_{r=1}^{n} u_r$ If $\lim_{n\to\infty} S_n = l$ (finite), then the series $\sum_{n=1}^{\infty} u_n$ is said to be convergent and the sum to infinity is l. i.e. $\sum_{n=1}^{\infty} u_n = l$ If S_n does not tend to a limit, then $\sum_{n=1}^{\infty} u_n$ is said to be divergent. Discuss the convergence of an infinite geometric series. In a geometric series with first term a and common ratio r, the series is convergent if $\lvert r\rvert < 1$ and the sum to infinity is $\frac{a}{1-r}$	

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
		Centre of Mass (Gravity)	
2.11	1. Defines the centre of mass of a system of particles in a plane.	Let the mass of the particle at $P_r \equiv (x_r, y_r)$ with respect to rectangular cartesian coordinate system chosen in the plane of a coplanar system of particles, be where $r = 1, 2, 3, \ldots n$. There exists a point $G \equiv (\bar{x}, \bar{y})$ in the plane of the system of particles such that, $\bar{x} = \dfrac{\sum_{r=1}^{n} m_r x_r}{\sum_{r=1}^{n} m_r}$ and $\bar{y} = \dfrac{\sum_{r=1}^{n} m_r y_r}{\sum_{r=1}^{n} m_r}$ G is called the centre of mass of the system of particles.	10
	2. Defines the centre of gravity of the system of particles in a plane.	The weight of a body is equal to the weights of its constituent particles and acts vertically downward through a fixed point in the body. The fixed point is called the centre of gravity where the fixed point is independent of the orientation of the body.	
	3. Defines the centre of mass of a lamina.	Let a tiny mass at the point $P \equiv (x, y)$ with respect to a cartesian system of coordinates chosen in the lamina be δm. The point $G \equiv (\bar{x}, \bar{y})$ is such that $\bar{x} = \dfrac{\int x\,dm}{\int dm}$ and $\bar{y} = \dfrac{\int y\,dm}{\int dm}$	
	4. Finds the centre of gravity of uniform bodies about a symmetrical line.	Bodies in which the masses are distributed with the same constant density are known as uniform bodies. 1. Centre of gravity of a thin uniform rod. 	

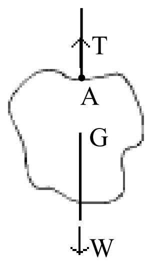
Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
		2. Centre of gravity of a uniform rectangular lamina. 	
		3. Centre of gravity of a uniform circular ring. 	
		4. Centre of gravity of a uniform circular disc. 	
	5. Finds the centre of gravity of a uniform lamina.	1. Centre of gravity of a uniform triangular lamina. Show that the centre of gravity of a triangle lies at the point of intersection of the medians - that is two thirds of the distance from each vertex to the midpoint of the opposite side. 2. Centre of gravity of a uniform parallelogram. Show that the centre of gravity of a parallelogram is the point of the intersection of its diagonals.	
	6. Finds the centre of gravity of bodies symmetrical about a plane.	Discuss the centre of gravity of the following uniform bodies. (i) hollow cylinder (ii) solid cylinder (iii) hollow sphere (iv) solid sphere	

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
2.12	1. Finds the centre of gravity of symmetrical bodies using integration.	When a body cannot be divided into a finite number of parts with known centres of gravity it may be divided into infinite number of parts with known centres of gravity. Summing the moments of the parts is done by integration. Show by integration that 1. the centre of gravityof a uniform circular arc of radius *a* subtending an angle 2α at the centre lies at a distance $\frac{a\sin\alpha}{\alpha}$ from the centre along the axis of symmetry. 2. The centre of gravityof *a* uniform circular sector of radius *a* subtending an angle 2α at the centre lics at a distance $\frac{2a\sin\alpha}{3\alpha}$ from the centre along the axis of symmetry. 3. Show that the centre of gravity of a solid hemisphere with radius *a* lies at a distance $\frac{3a}{8}$ from the centre along the axis of symmetry. 4. Show that the centre of gravity of a hollow hemisphere with radius *a* lies at a distance $\frac{a}{2}$ from plane face along the axis of symmetry. 5. Show that the centre of gravity of a uniform solid right circular cone of height *h* lies at a distance $\frac{h}{4}$ from base along the axis of symmetry. 6. Show that the centre of gravity of a uniform hollow cone of height *h* lies at a distance $\frac{h}{3}$ from base along the axis of symmetry .	08

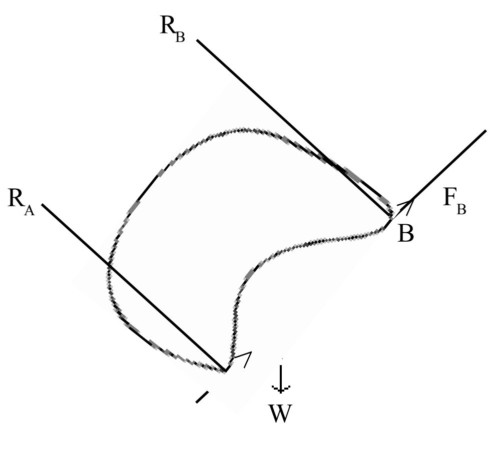
<table>
<tr><th>Com. Level</th><th>Learning Outcomes</th><th>Guidelines for Subject Matter</th><th>No. of Period</th></tr>
<tr><td></td><td>2. Finds the centre of gravity of bodies obtained by revolving.</td><td>Discuss the position of centre of gravity of a uniform solid formed by revolving the section of a curve.

Example:
$y^2 = 4ax$ revolving about x axis between $x = 0$ and $x = a$.</td><td></td></tr>
<tr><td>2.13</td><td>Finds the centre of gravity of composite bodies and remainders.</td><td>When a body is made up from two or more parts, each of which has a known weight and centre of gravity, then as the weight of the complete body is the resultant of the weights of its parts. We can use the principle of moments to find the centre of gravity of the body.

Discuss problems on composite bodies.
Similarly for remainders.</td><td>04</td></tr>
<tr><td>2.14</td><td>Explains the stability of bodies in equilibrium.</td><td>1. Hanging bodies:
Since there are only two forces acting on the body they must be equal and opposite.
i.e. T = W and
AG is vertical



2. Bodies resting on an inclined plane.


</td><td>04</td></tr>
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Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
		The forces acting on the body are (i) Weight (ii) Normal reactions R_A, R_B at the points of contact A & B respectively. (iii) Frictions at A & B. **For equilibrium:** The vertical through the centre of gravity must fall between A and B. If the vertical through G falls outside AB there is a turning effect and the body will topple. **Probability**	
4.1	1. Explains random experiment.	Discuss what is random experiment.	04
	2. Defines sample space.	The collection of all possible outcomes for an experiment is called sample space.	
	3. Defines an event.	An event is a subset of a sample space. i.e. An event is a collection of one or more of the outcomes of an experiment.	
	4. Explains evnt space.	Set of all events of a random experiment is said to be an event space.	
	5. Explains simple events and compound events.	An event that includes one and only one of the outcomes for an experiment is called a simple event. A compound event is a collection of more than one outcome for an experiment. **Explain** (i) Union of two events (ii) Intersection of two events (iii) Mutually exclusive events (iv) Exhaustive events	
4.2	1. States classical definition of probability and its limitations.	The probability of an event 'A' related to a random experiment consisting of N equally probable event is defined as $P(A) = \frac{n(A)}{N}$. Where $n(A)$ is the number of simple events in the event A.	04

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
		Limitations : (i) The above formulae cannot be used when the results of the random experiment are not equally probable. (ii) When the sample space is infinite the above formulae is not valid.	
	2. States the aximotic definition.	Let $\mathcal{E}$ be the event space corresponding to a sample space Ω of a random experiment. A function $P: \mathcal{E} \rightarrow [0,1]$ Satisfying the following conditions. (i) $P(A) \geq 0$ for any $A \subseteq \Omega$ (ii) $P(\Omega) = 1$ (iii) If A_1, A_2 are two mutually exclusive events $P(A_1 \cup A_2) = P(A_1) + P(A_2)$ is said to be a probability function.	
	3. Proves the theorems on probability using axiomatic definition and solves problems using the above theorems.	Prove that (i) $P(\phi) = 0$ (ii) $P(A') = 1 - P(A)$ (iii) $P(A) = P(A \cap B) + P(A \cap B')$ (iv) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (v) If $A \subseteq B$, then $P(A) \leq P(B)$	08
4.3	1. Defines conditional probability.	Let Ω be the sample space of a random experiment and A and B be two events where $P(A)>0$, then the conditional probability of the event B given that the event A has occured, denoted by $P(B/A)$, is defined as $P(B/A) = \dfrac{P(B \cap A)}{P(A)}$.	
	2. Proves the theorems on conditional probability.	Prove that, (i) If $P(A) > 0$, then $P(\phi/A) = 0$ (ii) If $A, B \in \mathcal{E}$ and $P(A) > 0$ then $P(B'/A) = 1 - P(B/A)$	

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
		(iii) If $A, B_1, B_2 \in \mathcal{E}$, then $P(B_1/A) = P(B_1 \cap B_2/A) + P(B_1 \cap B_2'/A)$	
	3. States multiplication rule.	$P(A_1 \cap A_2) = P(A_1).P(A_2/A_1)$ State multiplication rule for three events.	06
4.4	1. Defines independent events.	Let A_1, A_2 be two events on $\mathcal{E}$ and if A_1 and A_2 are independent then $P(A_1 \cap A_2) = P(A_1).P(A_2)$. Explain independence for three events.	06
4.5	1. Defines partition of a sample space.	Let $B_1, B_2, ..., B_n$ be events in the event space related to sample space Ω of a random experiment. $\{B_1, B_2, B_3, ..., B_n\}$ is said to be a partition of Ω if (i) $\bigcup_{i=1}^{n} B_i = \Omega$ (ii) $B_i \cap B_j = \phi \quad (i \neq j, 1 \leq i, j \leq n)$	
	2. States the theorem on total probability.	Let $\{B_1, B_2, ..., B_n\}$ be a partition of the event space corresponding to the sample space Ω. If $P(B_i) > 0$ and if A is any event in the event space $\mathcal{E}$, then $P(A) = \sum_{i=1}^{n} P(A/B_i)P(B_i)$.	
	3. Sates Baye's theorem and applies for problems.	Let $\{B_1, B_2, ..., B_n\}$ be a partition of the event space $\mathcal{E}$. If A is any event in then $P(B_j/A) = \dfrac{P(A/B_j)P(B_j)}{\sum_{i=1}^{n} P(A/B_i)P(B_i)}$	

Third Term

COMBINED MATHS I (3rd Term)

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
10.1	1. Explain Pascal triangle.	**Binomial Expansion** 1 1 2 1 1 3 3 1 1 4 6 4 1 • • • • • • This array of numbers, which is such that each number, except those at the ends, is the sum of the two numbers on either side of it in the line above known as Pascal Triangle.	06
	2. States and proves Binomial Theorem.	Statement of the theorem for positive integral index, $(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1} b$ $+ \ldots\ldots + {}^nC_r a^{n-r} b^r + \ldots\ldots + {}^nC_n b^n$, where ${}^nC_r = \dfrac{n!}{(n-r)!\,r!}$ for $0 \le r \le n$ Proof of the theorem, (i) Using Mathematical induction (ii) Using combinations	
	3. Explains the difference between coefficients and binomial coefficients of expansion.	In the expansion, $(a+x)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1} x + \ldots$ $+ {}^nC_r a^{n-r} x^r + \ldots + {}^nC_n x^n$, ${}^nC_0, {}^nC_1, \ldots, {}^nC_n$ are called binomial coefficients. ${}^nC_0 a^n, {}^nC_1 a^{n-1}, \ldots, {}^nC_n$ are the coefficients of the expansion. (i) The number of terms in the expansion is $(n+1)$ (ii) General term of the expansion is $T_{r+1} = {}^nC_r a^{n-r} x^r$. $(1+x)^n = \sum_{r=0}^{n} {}^nC_r x^r$.	

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
	4. Finds the properties of Binomial coefficients.	Using the above expansion obtain the properties of binomial coefficients.	
10.2	Finds the greatest term and greatest coefficient in the Binomial expansion.	Discuss how to find the greatest term and the greatest coefficient in the Binomial expansion.	06
		Complex Numbers	
14.1	1. Identifies imaginary unit and pure imaginary numbers.	Introduce the imaginary unit *i* such that $i^2 = -1$. The numbers of the form *ai*, where a $\mathbb{R}$, are called pure imaginary numbers. Discuss i^n, $n \in \mathbb{Z}^+$	02
	2. Defines a complex number.	A complex number is defined as $z = a + ib$, where $a, b \in \mathbb{R}$ and $i^2 = -1$. *a* is called the real part of the complex number *z* and denoted by $\mathrm{Re}(z)$ and *b* is called the imaginary part of the complex number *z* and denoted by $\mathrm{Im}(z)$.	
	3. States the conditions for equality of two complex numbers.	If $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$ are two complex numbers, then $z_1 = z_2 \Leftrightarrow a_1 = a_2$ and $b_1 = b_2$.	
	4. Defines conjugate of a complex number.	If $z = a + ib$, then the complex conjugate of *z* (denoted $\bar{z}$) is defined as $\bar{z} = a - ib$.	
14.2	Defines algebraic operations on complex numbers.	Let $z_1 = a_1 + ib_1$, $z_2 = a_2 + ib_2$ and $\lambda \in \mathbb{R}$ Then (i) $z_1 + z_2 = (a_1 + a_2) + i(b_1 + b_2)$ (ii) $\lambda z = \lambda(a + ib) = \lambda a + i\lambda b$ (iii) $z_1 - z_2 = (a_1 - a_2) + i(b_1 - b_2)$ (iv) $z_1 z_2 = (a_1 a_2 - b_1 b_2) + i(a_1 b_2 + a_2 b_1)$ (v) $\dfrac{z_1}{z_2} = \dfrac{a_1 + ib_1}{a_2 + ib_2} =$ $\left(\dfrac{a_1 a_2 + b_1 b_2}{a_2^2 + b_2^2}\right) + i\left(\dfrac{a_2 b_1 - a_1 b_2}{a_2^2 + b_2^2}\right)$ for $z_2 \neq 0$	02

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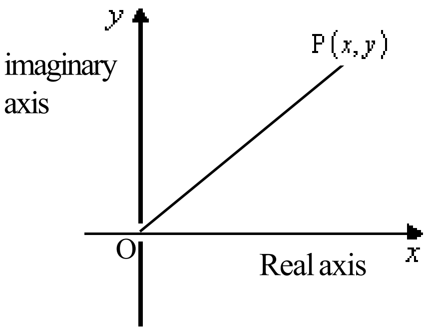
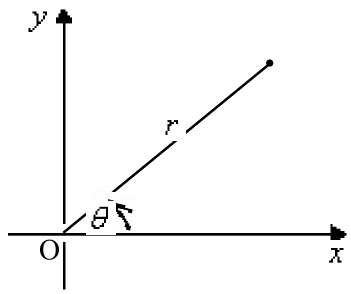
		The set of complex numbers is closed under the above operations. Show that $z+\bar{z}$ and $z\bar{z}$ are real numbers.	
14.3	1. Represents a complex number in Argand diagram.	Introduce Argand diagram (complex plane) Represents a complex number as a point in Argand diagram. Let $z = x + iy$ Then the point $\mathrm{P}(x, y)$ represents z in the Argand diagram. 	03
	2. Defines modulus of a complex number.	Modulus of the complex number z is denoted by $\lvert z \rvert$ $\mathbb{C}$ $\lvert z \rvert = \sqrt{x^2 + y^2} \geq 0$ $\lvert z \rvert = \mathrm{OP} = r$	
	3. Expresses a non zero complex number in $r(\cos\theta + i\sin\theta)$ form.	Let $z = x + iy$ be a non-zero complex number. Then $z = \sqrt{x^2+y^2}\left\{\frac{x}{\sqrt{x^2+y^2}} + i\frac{y}{\sqrt{x^2+y^2}}\right\}$ $= r(\cos\theta + i\sin\theta)$ where $r = \sqrt{x^2+y^2}$ $\cos\theta = \frac{x}{r}$ and $\sin\theta = \frac{y}{r}$	
	4. Defines argument of a complex number.	Let z be a non-zero complex number. An angle θ satisfying $z = r(\cos\theta + i\sin\theta)$ is called an argument of z.	

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
	5. Defines arg z.	Let z be a non-zero complex number. The set of all values of θ for which $z = r(\cos\theta + i\sin\theta)$ is denoted by arg z.	
	6. Defines Arg z.	Let z be a non-zero complex number. The value of θ for which $z = r(\cos\theta + i\sin\theta)$, where $-\pi < \theta \leq \pi$ is denoted by Arg z. Arg z is called the principal value of the argument.  	
	7. Construct points in the Argand Diagram.	Given z, construct the points representing. (i) λz (ii) $\bar{z}$; $\lambda \in \mathbb{R}$ Given two complex numbers z_1, z_2, construct the points representing. (i) $z_1 + z_2$ (ii) $z_1 - z_2$ (iii) $\dfrac{\lambda z_1 + \mu z_2}{\lambda + \mu}$ where $\lambda, \mu \in \mathbb{R}$ in the Argand Diagram. Obtain the triangle inequality $\lvert z_1 + z_2 \rvert \leq \lvert z_1 \rvert + \lvert z_2 \rvert$ for $z_1, z_2 \in \mathbb{C}$ Deduce that $\lvert \lvert z_1 \rvert - \lvert z_2 \rvert \rvert \leq \lvert z_1 - z_2 \rvert$ for $z_1, z_2 \in \mathbb{C}$	
14.4	1. Finds the modulus and argument of product of two complex numbers.	If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$ Show that $z_1 z_2 = r_1 r_2 \left[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\right]$ $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]$	05

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
	2. Constructs points representing $z_1 z_2$ and $\frac{z_1}{z_2}$ in the Argand, diagram.	Show the construction of $z_1 z_2$ and $\frac{z_1}{z_2}$ in Argand diagram. Given z, find the point represented by $z(\cos\alpha + i\sin\alpha)$	
14.5	1. Finds the loci on the complex plane.	Let the complex numbers z, z_0, z_1 and z_2 be represented by the points $\mathrm{P}, \mathrm{P}_0, \mathrm{P}_1, \mathrm{P}_2$ respectively. Show that (i) the locus of z given by $\lvert z - z_0 \rvert = r$ is a circle with centre P_0 and radius r. Obtain the cartesian equation of the locus. (ii) The locus of z given by the equation $\mathrm{Arg}(z - z_0) = \alpha$ is the half line PP_0 which make an angle α with positive direction of x-axis. (iii) The locus of z given by the equation $\lvert z - z_1 \rvert = \lvert z - z_2 \rvert$ is the line which is perpendicular bisector of $\mathrm{P}_1\mathrm{P}_2$ and obtain the cartesian equation of the line.	04
		Matrices	
12.1	1. Defines a matrix.	Matrix is a rectangular array of numbers. Matrices are denoted by capital letters A, B, C.. etc. $\mathrm{A} = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$ If A has m rows and n columns the order of the matrix A is $m \times n$. A is also written as $(a_{ij})_{m\times n}$ Element of a matrix : a_{ij} is the element of i^{th} row and j^{th} column. *Row matrix :* A matrix which has only one row is called a row matrix or a row vector.	02

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
		Column matrix: A matrix which has only one column is called a column matrix or column vector. Null matrix: If every element of a matrix is zero, it is called a null matrix.	
	2. Defines the equality of matrices.	If two matrices $A = (a_{ij})_{m\times n}$ and $B = (b_{ij})_{m\times n}$ are of the same order and if $a_{ij} = b_{ij}$ for $i = 1, 2, 3, \ldots, m$ and $j = 1, 2, 3, \ldots, n$ then A = B	
	3. Defines the addition of matrices.	State the conditions. (i) Matrices are of the same order, (ii) Corresponding element are added. Addition is Commutative and Associative	
	4. Defines the multiplication of a matrix by a scalar.	If $A = (a_{ij})_{m\times n}$; $\lambda \in \mathbb{R}$, then $\lambda A = (\lambda a_{ij})_{m\times n}$ for $i = 1, 2, 3, \ldots, m$ and $j = 1, 2, 3, \ldots, n$ When $\lambda = -1$ $(-1)A$ is denoted by $-A$.	
	5. Defines the transpose of a matrix.	Transpose of a matrix A is denoted by A^T. Let $A = (a_{ij})_{m\times n}$ Then $A^T = (b_{ij})_{n\times m}$, where $b_{ij} = a_{ji}$ for $1 \le i \le m$ and $1 \le j \le n$ $(A+B)^T = A^T + B^T$ $(A^T)^T = A$	

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12.2	1. Explains special cases of matrices.	Define a square matrix. If $m = n$ in a matrix A of order $m \times n$, then A is called a square matrix of order n. $$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}$$ $(a_{11}, a_{22}, a_{33}, \ldots, a_{nn})$ is the leading (Principal) diagonal. • A square matrix A is said to be an identity $a_{ij} = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ if } i \neq j \end{cases}$ and denoted by I_n • A square matrix A is said to be diagonal if $a_{ij} = 0$ for all $i \neq j$. • A square matrix A is said to be symmetric if $A^T = A$. • A square matrix A is said to be skew symmetric if $A^T = -A$ • A square matrix A is said to be upper triangular matrix if $a_{ij} = 0$ when $i > j$. • A square matrix A is said to be lower triangular matrix if $a_{ij} = 0$ when $i < j$.	01
12.3	1. Defines the multiplication of matrices.	Let $A_{m \times p}$ and $B_{q \times n}$ be two matrices. Multiplication AB is compatible when $p = q$. If $A = (a_{ij})_{m \times p}$ and $B = (b_{ij})_{p \times n}$ under compatibility, $$AB = \left[\sum_{k=1}^{p} [a_{ik} b_{kj}]\right]_{m \times n}$$ is of order $m \times n$ Discuss that even when AB is defined, BA is not necessarily defined. In general $AB \neq BA$.	04

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	2. Uses theorems in solving problems.	For square matrices A, B and C of same order *n*. (i) $(\lambda A)B = \lambda(AB) = A(\lambda B),\ \lambda \in \mathbb{R}$. (ii) $A(BC) = (AB)C$ (associative) (iii) $A(B+C) = AB+AC$ (distributive) (iv) $(B+C)A = BA+CA$ (distributive) (v) $A \times O = O = O \times A$ (O is the zero matrix) (vi) $AI_n = A = I_nA$ (vii) $(AB)^T = B^TA^T$ (viii) AB = O does not necessarily follow that A = O or B = O Let $P(x) = \sum_{i=0}^{m} a_i x^i$ and A be a square matrix of order *n* then $P(A)$ is given by $P(A) = \sum_{i=0}^{m} a_i A^i$ where $A^0 = I_n$	
	3. Finds the inverse of 2×2 matrix.	Find the value of a 2×2 determinant. Given a matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ determinant of A denoted by det A or $\lvert A \rvert$ is defined as det A = $\lvert A \rvert = ad - bc$. State that, given a square matrix A, if there exists a matrix B such that $AB = I_2 = BA$, then B is said to be the inverse of A and denoted by A^{-1}. Therefore, $AA^{-1} = I_2 = AA^{-1}$ Show that (i) $(A^{-1})^{-1} = A$ (ii) $(A^{-1})^T = (A^T)^{-1}$ (iii) $(AB)^{-1} = B^{-1}A^{-1}$ Given that $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $\lvert A \rvert \neq 0$, show	

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
		that $A^{-1} = \frac{1}{\lvert A \rvert}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ Discuss the inverses for diagonal matrices, upper triangular matrices and lower triangular matrices of order 3.	
12.4	Solves simultaneous equations using matrices.	Given that $a_1x + b_1y = c_1$ $a_2x + b_2y = c_2$ write the above equations in the form AX = C, where $A = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \end{pmatrix}$ and $C = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$ If A^{-1} exists $A^{-1}AX = A^{-1}C$ $(A^{-1}A)X = A^{-1}C$ $X = A^{-1}C$ Discuss solutions of simultaneous equations to illustrate the following situations. (i) Unique solution (ii) Infinite number of solutions (iii) No solution.	06
13.1	1. Defines the value of a determinant.	**Determinants** (i) State the forms of 2×2 and 3×3 determinants. (ii) Value of 2×2 determinant If $\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$, then $\Delta = a_1b_2 - a_2b_1$ (iii) Value of a 3×3 determinant $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ $= a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$ $\Rightarrow \Delta = a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2)$ $+ c_1(a_2b_3 - a_3b_2)$	08

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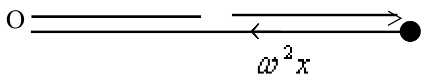
	2. Defines the minor of an element in a 3×3 determinant.	If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and the minor of the element in i^{th} row and j^{th} column, denoted by M_{ij} is the 2×2 determinant obtained by deleting i^{th} row and j^{th} column of .	
	3. Defines the cofactor of an element in a 3×3 determinant.	If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$, then cofactor of the element a_{ij} denoted as A_{ij} is given by $A_{ij} = (-1)^{i+j}\, M_{ij}$, where $i, j = 1, 2, 3$	
	4. States the properties of determinants.	Discuss and verify the following properties. (i) Let A be a square matrix of order 3. Then $\det A = \det A^T$ (ii) If all the elements in a row (columns) are zero, the value of determinant is zero. (iii) If any two rows (column) are interchanged, the determinant changes its sign. (iv) The value of a determinant is unaltered if a multiple of any row (column) is added to any other row (column). (v) If one row (column) of a determinant (Δ) is multiplied by a scalar λ, the resulting determinant is equal to $\lambda\Delta$. (vi) If $\Delta = \begin{vmatrix} x_1 & y_1 & a_1 \pm b_1 \\ x_2 & y_2 & a_2 \pm b_2 \\ x_3 & y_3 & a_3 \pm b_3 \end{vmatrix}$ $\Delta_1 = \begin{vmatrix} x_1 & y_1 & a_1 \\ x_2 & y_2 & a_2 \\ x_3 & y_3 & a_3 \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} x_1 & y_1 & b_1 \\ x_2 & y_2 & b_2 \\ x_3 & y_3 & b_3 \end{vmatrix}$ then $\Delta = \Delta_1 \pm \Delta_2$	

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
	5. Uses determinants to find the solutions of simultaneous equations.	Discuss the solutions of simultaneous equations in two variables. Let $a_1x+b_1y = c_1$ $a_2x+b_2y = c_2$ Using Cramer's rule: $x = \dfrac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$ and $y = \dfrac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$, provided $a_1b_2 - a_2b_1 \neq 0$ Discuss the solutions for three unknowns. Let $a_1x+b_1y+c_1z = d_1$ $a_2x+b_2y+c_2z = d_2$ $a_3x+b_3y+c_3z = d_3$ Using Cramer's rule: $x = \dfrac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$, $y = \dfrac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$ and $z = \dfrac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$, provided $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$	

COMBINED MATHS II (3rd Term)

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
		Simple Harmonic Motion	
3.16	1. Defines Simple Harmonic Motion (SHM).	State that Simple Harmonic Motion is a particular type of oscillatory motion. • It is defined as a motion of a particle moving in a straight line with a linear acceleration proportional to the linear displacement from a fixed point and is always directed towards that fixed point. • The fixed point is known as the centre of oscillation.	06
	2. Obtains the differential equation of Simple Harmonic Motion and verifies its general solutions.	 $\ddot{x} = -\omega^2 x$ The above is the differential equation for linear SHM, where ω is a constant. • Verify that $x = A\cos\omega t + B\sin\omega t$ is the general solution of the above differential equation, where A, B are arbitrary constants and t is the time.	
	3. Obtains the velocity as a function of displacement.	Discuss $x = A\cos\omega t + B\sin\omega t$ implies that $\dot{x}^2 = \omega^2\left[\left(A^2+B^2\right)-x^2\right]$ $\Rightarrow \dot{x}^2 = \omega^2\left[a^2 - x^2\right]$, where $a^2 = A^2 + B^2$ For the displacement, the following formulae can also be used. $x = a\sin\left(\omega t + \alpha\right)$	
	4. Defines amplitude and period of SHM.	State that (i) The length $a = \sqrt{A^2+B^2}$ is the amplitude of the SHM. (ii) The time $T = \frac{2\pi}{\omega}$ is the period of the SHM.	

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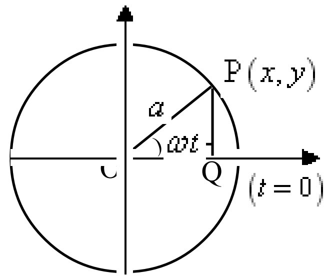
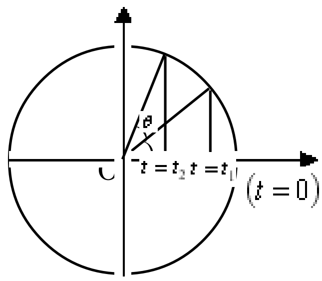
	5. Describes SHM associated with uniform circular motion.	Discuss  $x = a\cos\omega t$ $\dot{x} = -a\omega\sin\omega t$ $\ddot{x} = -a\omega^2\cos\omega t = -\omega^2 x$ Let a particle P moves in a circular motion with uniform angular velocity ω . Let Q be the foot of the perpendicular from P on a diameter .When P describes the circular motion, Q describes a SHM given by the equation $\ddot{x} = -\omega^2 x$.	
	6. Finds time duration between two position.	 Discuss the time duration between two positions of the particle. $t_2 - t_1 = \frac{\theta}{\omega}$ The above time interval can also be derived from the equations of the motion.	
3.17	Describes the nature of Simple Harmonic Motion on a horizontal line.	State Hookes law for tension or thrust. $T = \lambda\frac{d}{l}$, where λ : modulus of elasticity d : extension or compression l : natural length Prove by integration that the elastic potential energy is $\frac{\lambda d^2}{2l}$.	06

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		Discuss the Simple Harmonic Motion of a particle under the action of elastic forces along a horizontal line.	
3.18	Explains Simple Harmonic Motion of a particle in a vertical line.	• Simple Harmonic Motion of a particle in a vertical line under the action of elastic forces and its own weight. • Combination of SHM and a free motion under gravity.	06
		Statistics	
5.1	1. Explains what is statistics.	State that statistics is the science of obtaining and analyzing quantitative data with a view to make inferences and decisions. • A statistic refers to a summary figure computed from a data set.	01
	2. Explains the nature of statistics.	Statistics can be divided into two areas. (i) Descriptive statistics (ii) Inferential statistics **Descriptive statistics** consists of methods for organizing displaying and describing data by using tables, graphs and summary measures. **Inferential statistics** consists of methods that use sample results to help make decisions or predictions about a population.	
5.2	1. Explains to obtain information from data.	State that data is a collection of facts or figures related to a variate. State that information as the manipulated and processed form of data. Data is used as input for processing and information the output of this processing.	01
	2. Explains what is experiment.	Discuss experiment as an activity to obtain data. Discuss the types of experiment.	
	3. Explains the types of data.	• State that discrete data is a variable whose values are countable. A discrete data can assume only certain values with no intermediate values.	

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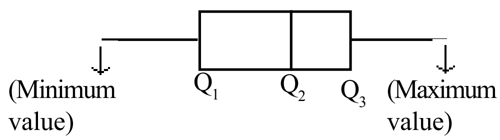
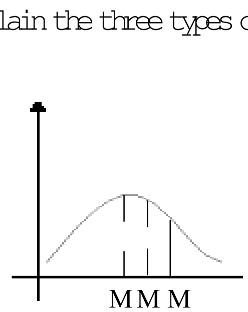
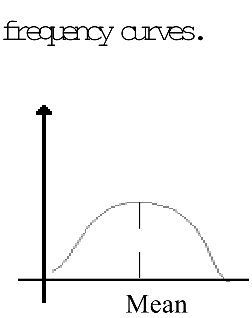
		• Continuous data is a variable that can take any numerical value over a certain interval.	
5.3	Classifies data and Information.	Discuss about the classification of data such as an array frequency distribution and stem-leaf.	01
5.4	Tabulates data and information.	Discuss the tabulation techniques for (i) Ungrouped frequency distribution (ii) Grouped frequency distribution (iii) Cumulative frequency distribution	01
5.5	Denotes data and information graphically.	Discuss the following graphical methods to denote. **(i) Bar Chart** A graph made of bars whose heights represents the frequencies of respective categories is called bar chart. **(ii) Pie chart** A circle divided into sectors that represents the relative frequencies or percentages of the categories they represent. **(iii) Histogram** Histogram is a bar chart without gaps in which the area of the bar is proportional to the frequency of the particular class. **(iv) Line graph** Line graphs consists of vertical lines, the height of a line represents the frequency of an ungrouped discrete data. **(v) Box plot** A box that shows three quartiles and whiskers extends from the box to the minimum and maximum values. The box represents the central 50% of the data. 	04

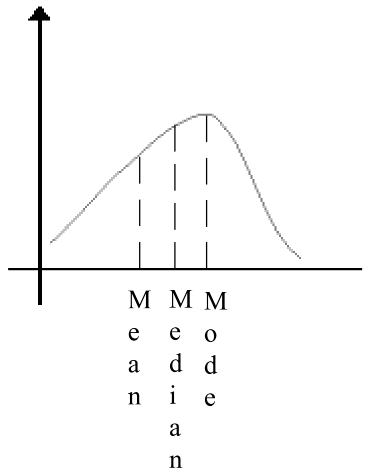
Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
5.6	Describes the mean, median and mode as measures of central tendency.	State mean, median and mode are the measures of central tendency for a set of data. The mean $\overline{x}$ of a set of data $x_1, x_2, \ldots, x_n$ is defined by $\overline{x} = \dfrac{\sum_{i=1}^{n} x_i}{n}$. Let $x_1, x_2, \ldots, x_n$ be a set of data with frequencies $f_1, f_2, \ldots, f_n$ respectively. Mean (Arithmetic mean) of a grouped data is defined as The mean $\overline{x} = \dfrac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$. (for grouped data x_i denotes the mid point of the i^{th} class interval) Discuss coding method. Discuss weighted mean: $\overline{x} = \dfrac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$, where w_i is the weight of x_i. ***Mode:*** State mode as a value of a variable which has the greatest frequency in a set of data. Mode may have more than one value. For a grouped frequency distribution, mode is given by Mode = $L_{mo} + c\left(\dfrac{\Delta_1}{\Delta_1 + \Delta_2}\right)$, where L_{mo} is the lower boundary of the modal class, c is the size of the class interval, $\Delta_1 = f_{mo} - f_{mo-1}$,	01

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
		$\Delta_2 = f_{mo} - f_{mo+1}$ and f_{mo} is the frequency of the modal class.	
5.7	Interprets frequency distribution using relative positions.	Median is the middle value of an ordered set of data. 1. Let $x_1, x_2, \dots, x_n$ be the ordered set of n data. Median is the $\left(\frac{n+1}{2}\right)^{th}$ value of the ordered set. Discuss the cases (i) When n is odd, (ii) When n is even. 2. Discuss for ungrouped frequency distribution also. 3. For a grouped frequency distribution f_c Median $= b + \frac{\left(\frac{N}{2} - f\right)c}{f_c}$, where b is the lower class boundary of the median class c is the size of the class interval f is the sum of all frequencies below b and is the frequency of the median class. ***Quartiles:*** **First Quartile (Q_1):** Q_1 is the $\left(\frac{n+1}{4}\right)^{th}$ value of the data arranged in the ascending order. **Second Quartile (Q_2):** Q_2 is the $\left(\frac{n+1}{2}\right)^{th}$ value of the data arranged in the ascending order.	04

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
		Third Quartile (Q_3): Q_3 is the $\frac{3}{4}(n+1)^{th}$ value of the data arranged in the ascending order. Note that Q_2 (median) is the 2^{nd} Quartile. Note: Discuss ungrouped frequency distributions and grouped frequency distributions with examples **Percentile :** p^{th} percentile of the data is given by $\left(\frac{pn}{100}\right)^{th}$ value of the data arranged in the ascending order . Discuss the integer as well as the non-integer cases.	
5.8	Uses suitable measure of central tendency to make decisions on frequency distribution.	Discuss the uses of the measures of central tendency in frequency distributions. Explain with suitable examples.	04
5.9	Explains the measures of dispersion.	Dispersion indicates the spread of data. Measures of dispersion are used to represent the spread within data. Define the following types of measures of dispersion. 1. Range : Range is the difference between the largest value and the smallest value. 2. Interquartile Range: Interquartile Range = $Q_3 - Q_1$ 3. Semi interquartile Range = $\frac{Q_3 - Q_1}{2}$ 4. Mean Deviation: For a set of data $x_1, x_2, \ldots, x_n$, Mean deviation = $\frac{\sum_{i=1}^{n} \lvert x_i - \bar{x} \rvert}{n}$. For a frequency distribution,	08

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
		Mean deviation = $\dfrac{\sum_{i=1}^{n} f_i \lvert x_i - \bar{x}\rvert}{\sum_{i=1}^{n} f_i}$. (for a grouped frequency distribution x_i is the mid value of the i^{th} class) 5. Variance: For a set of data $x_1, x_2, \ldots, x_n$, Variance = $\dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$. Show that variance = $\dfrac{\sum_{i=1}^{n} x_i^2}{n} - \bar{x}^2$. For a frequency distribution, Variance = $\dfrac{\sum_{i=1}^{n} f_i (x_i - \bar{x})^2}{\sum_{i=1}^{n} f_i}$ (for grouped freqency distribution, x_i is the mid value of the i^{th} class). Show that variance = $\dfrac{\sum_{i=1}^{n} f_i x_i^2}{\sum_{i=1}^{n} f_i} - \bar{x}^2$. 6. Standard Deviation: Standard Deviation = $\sqrt{Variance}$ Let $\bar{x}$ be the mean and σ_x be the standard deviation for a set of data. Consider the linear transformation $y = ax + b$, where a and b are constants. Show that $\bar{y} = a\bar{x} + b$ and $\sigma_y = \lvert a\rvert \sigma_x$	

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
		7. Z – score Let $\bar{x}$ be the mean and σ_x be the standard deviation for a set of data $x_1, x_2, \ldots, x_n$. For each x_i, z_i is defined as $z_i = \dfrac{x_i - \bar{x}}{\sigma_x}$ z_i is called z-score of x_i. For the set of data $z_1, z_2, \ldots, z_n$ show that the mean is zero and the standard deviation is one. 8. Pooled mean (Combined mean) Let $\bar{x}_1$ and $\bar{x}_2$ be the means of sets of data with sizes n_1 and n_2 respectively. Show that the pooled mean $\bar{x} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$. Let σ_1^2 and σ_2^2 be the variances of sets of data with sizes n_1 and n_2 respectively. Show that the pooled variance $\sigma^2 = \dfrac{1}{n_1 + n_2}\left\{n_1\sigma_1^2 + n_2\sigma_2^2\right\} + \dfrac{n_1 n_2}{(n_1 + n_2)^2}\left(\bar{x}_1 - \bar{x}_2\right)^2$	
5.10	Determines the shapes of the distribution.	Explain the three types of frequency curves.  (Positively skewed)  (Symmetric)	02

Com. Level	Learning Outcomes	Guidelines for Subject Matter	No. of Period
		 (Negatively skewed) Pearson's coefficient of skewness is defined by $$K = \frac{\text{Mean} - \text{Mode}}{\text{Standard deviation}}$$ or by $$K = \frac{3(\text{Mean} - \text{Median})}{\text{Standard deviation}}$$	

School Based Assessment

Introduction- School Based Assessment

Learning –Teaching and Evaluation are three major components of the process of Education. It is a fact that teachers should know that evaluation is used to assess the progress of learning- teaching process. Moreover, teachers should know that these components influence mutually and develop each other .According to formative assessment (continuous assessment) fundamentals; it should be done while teaching or it is an ongoing process. Formative assessment can be done at the beginning, in the middle, at the end and at any instance of the learning teaching process.

Teachers who expect to assess the progress of learning of the students should use an organized plan. School based assessment (SBA) process is not a mere examination method or a testing method. This programme is known as the method of intervening to develop learning in students and teaching of teachers. Furthermore, this process can be used to maximize the student's capacities by identifying their strengths and weaknesses closely.

When implementing SBA programmes, students are directed to exploratory process through Learning Teaching activities and it is expected that teachers should be with the students facilitating, directing and observing the task they are engaged in.

At this juncture students should be assessed continuously and the teacher should confirm whether the skills of the students get developed up to expected level by assessing continuously. Learning teaching process should not only provide proper experiences to the students but also check whether the students have acquired them properly. For this, to happen proper guiding should be given.

Teachers who are engaged in evaluation (assessment) would be able to supply guidance in two ways. They are commonly known as feed-back and feed- forward. Teacher 's role should be providing Feedback to avoid learning difficulties when the students' weaknesses and inabilities are revealed and provide feed-forward when the abilities and the strengths are identified, to develop such strong skills of the students.

Student should be able to identify what objectives have achieved to which level, leads to Success of the Learning Teaching process. Teachers are expected to judge the competency levels students have reached through evaluation and they should communicate information about student progress to parents and other relevant sectors. The best method that can be used to assess is the S B A that provides the opportunity to assess student continuously.

Teachers who have got the above objective in mind will use effective learning, Teaching, evaluation methods to make the Teaching process and learning process effective. Following are the types of evaluation tools student and, teachers can use. These types were introduced to teachers by the Department of Examination and National Institute of Education with the new reforms. Therefore, we expect that the teachers in the system know about them well

Types of assessment tools:

1.	Assignments	2.	Projects
3.	Survey	4.	Exploration
5.	Observation	6.	Exhibitions
7.	Field trips	8.	Short written
9.	Structured essays	10.	Open book test
11.	Creative activities	12.	Listening Tests
13.	Practical work	14.	Speech
15.	Self creation	16	Group work
17.	Concept maps	18.	Double entry journal
19.	W all papers	20.	Quizzes
21.	Question and answer book	22.	Debates
23.	Panel discussions	24.	Seminars
25.	Impromptus speeches	26.	Role-plays

Teachers are not expected to use above mentioned activities for all the units and for all the subjects. Teachers should be able to pick and choose the suitable type for the relevant units and for the relevant subjects to assess the progress of the students appropriately.The types of assessment tools are mentioned in Teacher's Instructional Manuals.

If the teachers try to avoid administering the relevant assessment tools in their classes there will be lapses in exhibiting the growth of academic abilities, affective factors and psycho-motor skills in the students

Term 1

Group Assignment 1

01.1 Competency 29 : Interprets equations of Conics.

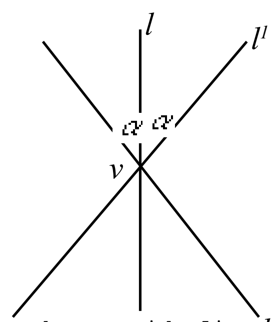
01.2 Nature of Group Assignment : An Assignment leading to the identification of conics as conic sections.

01.3 Instructions for the teacher

1. Present the assignment to the students 3 weeks before beginning the lesson on conics.
2. Instruct them to finish the assignment one week before beginning the lesson.
3. Evalute the finished assignment. Begin the lesson on conics on the scheduled date from their level of knowledge about conics.

Conics

Introduction:



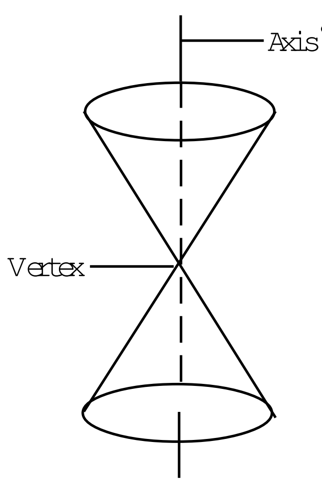
The solid of revolution generated in space by a straight line l passing through a fixed point v on a fixed straight line l and rotating at a constant acute angle with it is known as a cone.

v is known as the vertex of the cone, as the semi-vertical angle and as the generating line.

Each part of the cone separated through its vertex is called a nappe. Each such part is infinite (In practice a cone is known as a finite part of a nappe.)

Assignment

(1) Prepare 5 models as shown in the figure using a soft variety of timber or any other such material. Let them be named as A, B, C, D and E.



(2) (i) Separate the model A into two halves by means of a plane through its axis.

(ii) Separate one nappe of B by means of a plane perpendicular to its axis.

(iii) Separate one nappe of C by means of a plane which is neither parallel nor perpendicular to its axis and not parallel to a generating line.

(iv) Separate one nappe of D by means of a plane parallel to a generating line.

(v) Separate both nappes of E through the same plane.

(3) For each of the above cases trace the shape of edge / edges of the cutting section on a sheet of paper .

Name the curves obtained in (i), (ii), (iii) and (iv) above.

The curve consisting of two parts obtained in (v) above is known as a hyperbola.

(4) Write down the occasions where you have come across the curves mentioned above.

(5) (i) What is the conic section when the cone is cut by a plane passing through the centre only ?

(ii) What is the curve common to the plane through a generating line and the cone ?

Criteria for Evaluation

1. Finish of the given instrument.
2. Correctly obtained cutting sections.
3. Identifying the curves obtained from cutting sections.
4. Revealing practical situations.
5. Engaging in the activity as a group.

Term 1

Assignment 2

02.1 Competency 03.12 : Interprets the result of an impulsive action.

02.2 Nature of Group

Assignment : A group activity for the use of the principles of conservation of Linear Momentum and Conservation of Mechanical Energy.

02.3 Instructions for the teacher

1. Give this assignment to students to test whether the relevant concepts have been instilled in them after the lessons on impulse and simple momentum.
2. Give the necessary feed-back after evaluating the assignment.

Assignment

1. (i) A gun of mass M rests on a smooth horizontal plane and its barrel is inclined of an angle to the horizontal. It fires a bullet of mass m.

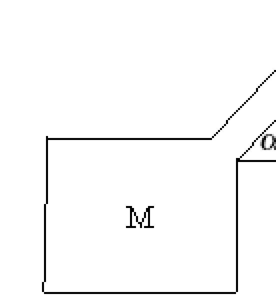


Fig. a

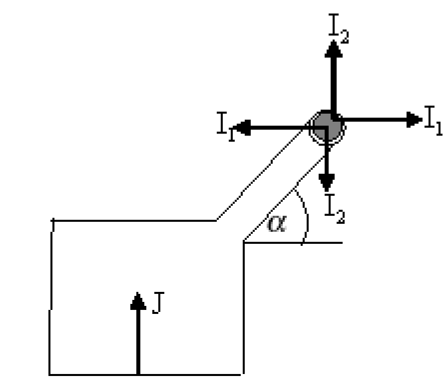


Fig. b

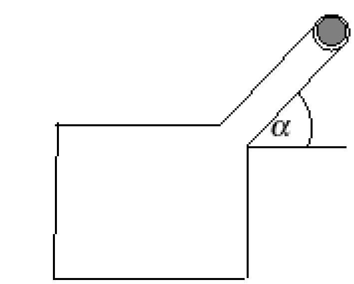


Fig. c

(ii) A small sphere of mass M is suspended by a light inextensible string and is at rest. Another small sphere of mass m and falling vertically downwards with a velocity u falls on M. At the moment of impact the line joining the centres of the spheres is inclined at an angle to the vertical.

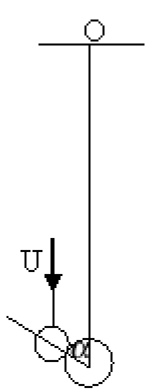


Fig. a

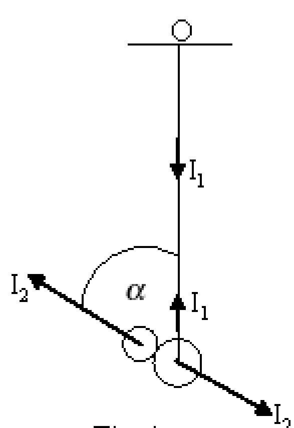


Fig. b

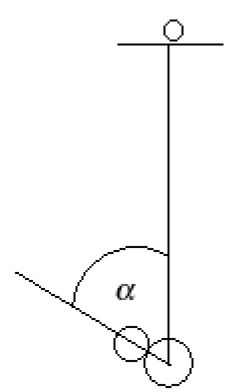


Fig. c

Fig (a) shows the instant just before the impact.

Fig (b) denotes the impulse created in the system.

In fig. (c) mark the velocities of M and m just after the application of the impulse.

To solve this problem in which direction should the principle of conservation of momentum be applied to the system ?

Write an equation by applying the principle of conservation of momentum in that direction.

2. A wedge of mass M rests on a smooth horizontal table. A particle of mass m is placed at the lower end of its face inclined at an angle to the horizontal and is projected with a velocity u up the face of the wedge so that it just reaches the vertex of the wedge.

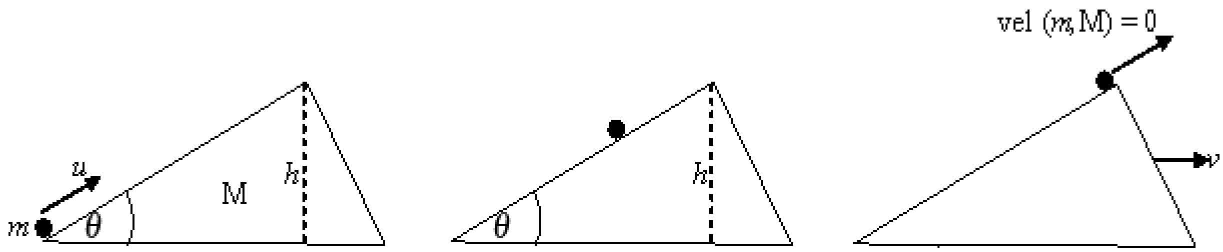


Fig. a Fig. b Fig. c

Figure (a) denotes the initial situation.

In figure (b) mark the forces acting on m and M.

Figure (c) denotes the situation when m reaches the vertex of M.

Explain why the principles of conservation of momentum and the conservation of energy can be applied in order to interpret this motion.

Derive two equations by applying those principles and by solving them show that $u^2 = \dfrac{2gh(M+m)}{M + m\,Sin^2\theta}$

3. One end of a light elastic string with natural length l and modulus of elasticity mg is attached to a fixed point O and a particle of mass m is attached to its other end. The particle m is projected vertically upwards with a velocity u from the point O. Answer the following questions to find the maximum length of the string in the subsequent motion.

(i) Draw a diagram for the initial position and mark the velocity of the particle.

(ii) Draw a diagram for a position when the string is unstretched in its motion above O and mark the forces acting on the particle.

(iii) Draw a diagram for a position when the string is stretched in its motion above O and mark the forces acting on the particle.

(iv) Draw a diagram for a position when the string is unstretched in its motion below O and mark the forces acting on the particle.

(v) Draw a diagram for a position when the string is stretched in its motion below O and mark the forces acting on the particle.

(vi) Draw a diagram for the position when the string has reached its maximum length and mark the velocity of the particle.

(vii) What can you say about the forces acting on the particle for the entire motion?

(viii) Explain why the principle of conservation of energy can be applied for the above motion?

(ix) Write an expression for the elastic potential energy (stored in) a string of modulus of elasticity λ and natural length l when it is stretched by a length x.

(x) For positions (i) and (vi) above write equations using the principle of conservation of energy.

(xi) Deduce the maximum length of the string.

Criteria for Evaluation

1. Understanding the principle of conservation of momentum.
2. Understanding the principle of conservation of energy.
3. Free expression of ideas.
4. Use of correct principles suitably.
5. Getting engaged in a task as instructed.

Straight Line

1.(a) The points P$(2, a)$ and Q$(b+1, 3b-2)$ both lie on the line $y = 5x + 1$. Find

(i) the values of a and b.

(ii) the distance between the points P and Q.

(b) The points A, B and C have co ordinates (6, –9) (–1, 15) and (–10, 3) respectively. Show that $\angle BCA = 90^0$ and hence calculate the cosine of $\angle BAC$

2. (a) The vertices of triangle ABC are A (2,5), B(2, –1) and C (–2, 3).

(i) Prove that for all values of t, the points with co-ordinates $(t-1, t)$ are equidistance from B and C.

(ii) Given that the point D is equidistant from A, B and C, calculate the co-ordinates of D.

(b) The line thorugh (2, 5) with gradient 3 cuts the x axis at A and y axis at B. Calculate the area of the triangle AOB, where O is the origin.

3.(a) Find the co-ordinates of the point where the line through (–3, 13) and (6, 10) cuts the line through (1, 5) with gradient 3.

(b) The centre of a square is at (3, 4) and one of its vertices at (7, 1). Find the co-ordinates of the other vertices of the square.

4.(a) Two vertices of a triangle are (5, –1) and (–2, 3). If the orthocentre of the triangle is the origin, find the co-ordinates of the third vertex.

(b) Find the equation of the bisector of the acute angle between the lines $3x - 4y + 7 = 0$ and $12x + 5y - 2 = 0$

5. (a) Two adjacent sides of a parallelogram are $2x - y = 0$ and $x - 2y = 0$. If the equation of one diagonal is $x + y = 6$. Find the equation of the other diagonal.

(b) Find all the points on the line $x + y = 4$ that lie at a unit distance from the line $4x + 3y = 10$.

Circle

1. (a) Write down the co-ordinates of the centre and radius of the circle with equation.

(i) $x^2 + y^2 + 6x + 4y - 36 = 0$ (ii) $2x^2 + 2y^2 - 2x - 2y - 1 = 0$

(b) Show that the circle with equation $x^2 + y^2 - 2ax - 2ay + a^2 = 0$ touches both the x axis and the y axis. Hence show that there are two circles pass through the point (2, 4) and touch both the x axis and the y axis. Find the equation of the tangent to each circle at the point (2, 4).

2. (a) Given that O (0, 0), A (3, 2) and B (2, 1), find the equation for

(i) the circle that passes through O, A and B.

(ii) the circle on AB as a diameter.

(b) Let $x^2 + y^2 - 6x + 8y = 0$ be a circle and P be the point (4, 3).

(i) Show that the point P lies outside the circle.

(ii) Find the length of the tangents from P to S.

(iii) Find the equations of the tangents from P to S.

(iv) Find the equation of the chord of contact of the tangents from P to S.

3. (a) Show that the line with equation $2x - 3y + 26 = 0$ is a tangent to the circle with the equation $x^2 + y^2 - 4x + 6y - 104 = 0$

(b) Show that the circle with equation $x^2 + y^2 - 4x + 2y - 4 = 0$ and the line with equation $x - 2y + 1 = 0$ interesect. Find the equation of the circle passing through the points of intersection of the above circle and line, and origin.

4. (a) Given that the circles $x^2 + y^2 - 6x + 4y + 9 = 0$ and $x^2 + y^2 - 4y + C = 0$

(i) touch, find the values of C and verify your answer.

(ii) cut orthogonally, find the value of C.

(b) Prove that two circles can be drawn through the origin to cut the circle $x^2 + y^2 - x + 3y - 1 = 0$ orthogonally and touch the line $x + 2y + 1 = 0$ and find their equations.

Work, Power, Energy

1.(a) A block of mass 500 kg is raised a height of 10 m by a crane. Find the work done by the crane against the gravity.

(b) A train travels 6 km between two stations. If the resistance to motion averages 500 N, find the work done against the resistance.

(c) A cyclist pushes his bicycle 100 m up a hill inclined at $\sin^{-1}\left(\frac{1}{10}\right)$ to the horizontal. If the cyclist and the cycle weigh 800 N, find the work done by the cyclist against gravity. If the road resistance to motion is 50 N, find the total work done by the cyclist.

2. A train of mass 150 tonnes (1 tonne = 1000 kg) is ascending a hill of gradient 1 in 15. The engine is working at a constant rate of 300 kw and the road resistance to motion is 40 N per tonne.Find the maximum speed of the train. Now the train moves on a horizontal track and engine works at the same rate. If the resistance to motion is unchanged. Find the initial accleration of the train.

3. A car of mass 200 kg pulls a caravan of mass 400 kg along a level road. The resistance to motion of the car is 1000 N and the resistance to motion of the caravan is 100 N. Find the acceleration of the car and the caravan at the instant when their speed is 40 kmh^{-1} with the power output of the engine equal to 100 kW. Find also the tension in the coupling between the car and the caravan at this instant.

Impulse and Momentum

1. A sphere of mass 1 kg, moving at 8 ms^{-1} strikes directly a similar sphere of mass 2 kg which is at rest. If the coefficient of restitution (e) $\frac{1}{2}$ find

   (i) The velocities of the spheres after impact.

   (ii) The Impulse between the spheres.

   (iii) The loss in kinetic energy due to collision.

2. A sphere of mass m, moving along a smooth horizontal table with speed V, collides directly with a stationary sphere of same radius and of mass $2m$.

   (i) Obtain expressions for the speeds of the two spheres after the impact in terms of V.

   (ii) Find the coefficient of restitution e.

   (iii) If half of the kinetic energy is lost due to collision find the value of e.

3. Two small smooth spheres A and B of equal radius but of masses $3m$ and $2m$ respectively are moving to wards each other 30 that they collides directly. Immediately before the collision, sphere A has speed $4u$ and sphere B has speed u. The collision is such that sphere B experiences an impulse of magnitude 6 mcu, where c is a constant. Find

   (i) In terms of u and c, the speeds of A and B immediately after the collision.

(ii) The coefficient of restitution in terms of c.

(iii) The range of values of c for which such collision would be possible.

(iv) The values of c such that $\frac{9}{16}$ of the total kinetic energy would be destroyed by the collision.

4. Two small spheres of masses m and 2m are connected by a light inextensible string of length 2a. When the string is taut and horizontal, its mid point is fixed and the spheres are released from rest. The coefficient of restitution between the spheres is $\frac{1}{2}$.

(i) Show that the first impact brings the heavier sphere to rest.

(ii) Show that the second impact brings the lighter sphere to rest.

(iii) Find the velocity of each sphere immediately after the third impact.

Term 2

Group Assignment 1

03.1 Competency Level: 8.1` Uses various methods for counting.

03.2 Nature : Group Assignment.

03.3 Instructions for the teacher

1. Direct the students to get engaged in this investigation about a week before beginning the lesson on permutations and combinations.
2. Instruct students to present the results of the investigation two days before the date scheduled for the lesson.
3. Evaluate the results of the investigation.
4. Begin the lesson on permutation and combination on the scheduled date from the level of their knowledge on permutations.

Note: The terms Principle of counting, permutation, combination and factorial notation should be introduced only after the teacher began the lesson.

03.4 Work sheet

Consider the following phenomenon.

This is an incident that has occured about hundred years ago.

A group of 10 students of a certain school were used to patronise the same canteen daily to have their tea during the school interval. They were in the habit of sitting on the same ten chairs which were in a row. One day the owner of the canteen made the following proposal to them.

"Today your group is seated in this order . When you come here tomorrow you sit in a different order and likewise change your sitting order daily . You have exhausted all the different orders or sitting I will give you all your refreshments free of charge."

Do the following activity in order to inquire into the canteen owner's proposal mathematically .

(i) Take 5 pieces of equal square card boards and mark them as A, B, C, D and E as shown below:

A	B	C	D	E

(ii) Draw two squares a little bigger than the above squares on a sheet of paper in a row .

In how many different ways can the two squares marked A and B can be placed inside the two squares on the sheet of paper.

(iii)(a) Drawing three square in a row and using the cards A, B and C.

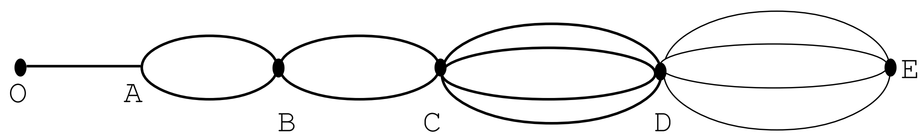
(b) Drawing four squares in a row and using the cards A, B, C and D.

(c) Drawing five squares in a row and using the cards A, B, C, D and E.

Find the number of different ways in which the cards can be placed with one card inside a square.

Note down the results of each of the cases above on a sheet of paper.

2. The network of a system of roads connecting the 5 cities A, B, C, D and E to a city O is as follows:



(a) In how many different ways can (i) A (ii) B (iii) C (iv) D (v) E can be reached from O?

(b) Describe a convenient way of obtaining the above results.

(c) Is there a relationship between these results and the results obtained in the activity (1) above.

If there is a relationship explain why it is so.

3. (a) Write an expression as a product of integers which gives the number of different ways in which 10 different objects (living, non-living or symbolic) can be placed in a row.

Simplify this expression. Hence write down your judgement with regard to the proposal made by the canteen owner mentioned earlier.

Write an expression in the form of a product for the number of different ways in which n different objects can be arranged in a row.

Criteria for Evaluation

1. Engaging in the task as instructed.
2. Revealing mathematical relationships.
3. Construction of mathematical models.
4. Reaching conclusions.
5. Expressing ideas logically.

Group Assignment 2

Nature of the student based activity: Open text assignment.

04.1 Competency Level:

04.1.1 Interprets the events of a random experiment.

04.1.2 Applies probability models for solving problems on random events.

04.2 Nature of the assignment : Open text assignment of revising the knowledge about sets and probability.

04.3 Instructions for the teacher

1. About 2 weeks before beginning the lesson probability instruct the students to study the lessons on sets and probability in the text books from grades 6 to 11. Distribute the given assignment to the students.
2. Instruct them to submit answers about one week before the beginning of the lesson.
3. After evaluation of the answers begin the lesson providing the necessary feedback.

Assignment

(1) (i) Write all subsets of A = {1, 2, 3, 4, 5 }. How many subsets are there ?

(ii) Select the subsets of B = $\{x \mid x \in \mathbb{Z}^+,\ x < 10\}$ from the following sets.

P = { 1, 4, 9, 16}	Q = { 2, 3, 5, 7 }
R = { Prime numbers less than 10 }	S = {Counting numbers less than 10}
T = { 2, 4, 6, 8 }	U = {1, 2, 3, 4, 5, 6, 7, 8, 9 }

Out of the subsets you have selected write down the proper subsets of A, if any.

(2) If A = {1, 2, 3, 4, 5 } , B = { 1, 3, 5, 7, 9 } and $\mathcal{E}$ = { 1, 2, 3, 4, 5, 6, 7, 8, 9}, write the elements of

(i) $A \cap B$ (ii) $A \cup B$ (iii) A' (iv) B' (v) $A' \cap B'$

(vi) $A' \cup B'$ (vii) $(A \cap B)'$ (viii) $A \cap B'$ (ix) $(A \cup B)'$ (x) $A' \cap B$

(3) State the following laws about set algebra and varify them by means of Venn diagrams.

(i) Commutative Law (ii) Distributive Law

(iii) Associative Law (iv) De Morgans Law

(4) Underline the correct results out of the folowing.

(i) $A \cap \phi = A$ (ii) $A \cup \phi = A$ (iii) $\mathcal{E} \cap \phi = A$

(iv) $A' \cup A = A$ (v) $A' \cap A = \phi$

(5) (i) Define a random experiment.

(ii) Select random experiments from the following:

(a) Sun will rise tomorrow.

(b) Testing the top side when a coin is tossed.

(c) Testing the top side when a dice marked from 1 to 6 is tossed.

(d) Testing the number of sick students sent home during school hours.

(e) Measuring the life span of an electric bulb.

(f) Drawing a ball at random from a bag containing 3 red balls and 1 blue ball which are identically equal.

(iii) Write the sample space of random experiments you have selected above.

(6) In the random experiment of observing the top sides when two coins are tossed simultaneously.

(i) Write the sample space.

(ii) Write two simple events in it.

(iii) Write two composite events in it.

(7) What are mutually exclusive events. Explain with an example.

(8) Toss a coin 25 times and complete the following table.

Number of Times	Side obtained (Head or Tail)
1	
2	
3	
.	
.	
.	
25	

(i) Find the success fraction of obtaining a head when the coin is tossed 25, times.

(ii) Repeat the experiment 50 times, 100 times and find the success fraction of obtaining a head.

(iii) If success fraction is to be taken as a measure of probability how should be the number of times the experiment is to be repeated?

(9) What is an equally probable event? Select equally probable events from the following random experiments.

(i) Observing the side obtained when a coin is tossed.

(ii) Observing the side obtained when an unbiased dice marked 1–6 is tossed.

(iii) Observing the colour of a ball taken randomly from a bag containing 2 blue balls and 3 red balls.

(iv) Observing the number of a card taken randomly from a set of identical cards numbered from 1–9.

(10) (i) Write the sample space for the random experiment (ii) above.

If A = { Obtaining an even number}

B = { Obtaining a prime number }

C = { Obtaining a square number }

D = { Obtaining an odd number }

(ii) Find

(a) P (A) (b) P (B) (c) P(C) (d) P(D) (e) $P(A \cap B)$

(f) $P(A \cap C)$ (g) $P(C \cap A)$ (h) $P(A \cup B)$ (i) $P(A \cup B \cup C)$ (j) $P(A \cap B \cap C)$

(iii) Prove that

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(b) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C)$
$- P(C \cap A) + P(A \cap B \cap C)$

(iv) (a) Select two mutually exclusive events.

(b) Find $P(A \cup D)$

Criteria for Evaluation

1. Use of text books for obtaining the necessary knowledge.
2. Knowledge of set Algebra.
3. Knowledge of basic concepts in probability.
4. Following the given instructions correctly.
5. Expressing ideas freely.

For the written test teacher can choose questions from the following or he / she can prepare questions on his/ her own.

Integration

1. (a) Express $\dfrac{2}{x(x+1)(x+2)}$ as the sum of partial fractions.

Hence show that $\displaystyle\int_2^4 \frac{2}{x(x+1)(x+2)}\,dx = 3\ln 3 - 2\ln 5$

(b) By using the substitution $x-1 = u^2$ or otherwise, find $\displaystyle\int \frac{x+1}{\sqrt{x-1}}\,dx$

(c) Use integration by parts to evaluate $\int_0^{\frac{\pi}{6}} x.\cos 3x dx$

(d) Show that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx$

2. (a) Express $\frac{1}{(3t+1)(t+3)}$ in partial fraction.

Use the substitution $t = \tan x$ to show that $\int_0^{\frac{\pi}{2}} \frac{1}{3+5\sin 2x} dx = \int_0^1 \frac{1}{(3t+1)(t+3)} dt$

Hence show that $\int_0^{\frac{\pi}{2}} \frac{1}{3+5\sin 2x} dx = \frac{1}{8}\ln 3$

(b) Use integration by parts to find $\int_0^1 x.e^{3x} dx$

(c) Find the area enclosed by the curve $y = x^2$ and $y^2 = x$

3 (a) By using the substitution $u^2 = a^2 - x^2$, or otherwise evaluate $\int_0^a x\sqrt{a^2 - x^2}\, dx$

(b) Let $f(x) = \frac{x^2 + 6x + 7}{(x+2)(x+3)}$

Find the values of constants A, B and C such that $f(x) = A + \frac{B}{x+2} + \frac{C}{x+3}$, and show that

$\int_0^2 f(x)dx = 2 + \ln\left(\frac{25}{81}\right)$

(c) Use integration by parts to find $\int_0^{\frac{\pi}{4}} x^2 \cos 2x\, dx$

(d) Show that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

Show that $\int_0^{\frac{\pi}{4}} \frac{1-\sin 2x}{1+\sin 2x}\, dx = \int_0^{\frac{\pi}{4}} \tan^2 x\, dx$ and evaluate $\int_0^{\frac{\pi}{4}} \frac{1-\sin 2x}{1+\sin 2x}\, dx$

Circular Motion

1. A light in-extensible string of length $14a$ has its ends attached to two fixed prints A and B. The point A is vertically above B and AB = $10a$. A particle of mass m is attached to the point P of the string, where AP = $8a$. The particle moves in a horizontal circle with angular speed w with the string taut

   (i) Show that the tension in AP is $\frac{4m}{25}(5g + 180w^2)$

   (ii) Find the tenam in BP

   (iii) Deduce that $w \geq \sqrt{\frac{5g}{32a}}$

2. A smooth wire is bent in the form of a circle of radius r and centre O and is fixed in a vertical plane. A bead B of mass m threaded on the wire is projected from the lowest point P with speed u.

   (i) Find the value of u if the bead first comes to rest when OP is horizontal.

   (ii) Find the least value of u with which the bead must be projected in order that the bead will move in complete circles.

   (iii) If $u = \sqrt{3ga}$ show that the reaction between the bead and wire is zero when OP makes an anble $\cos^{-1}\left(\frac{1}{3}\right)$ with upward vertical and find the angle OP makes with the vertical when the bead first becomes to rest.

3. A particle is slightly displaced from its position of rest on the top of a fixed smooth sphere of radius a.

   (i) Prove that it will leave the surface of the sphere when the angle between the radius through the particle and the vertical is $\cos^{-1}\left(\frac{1}{3}\right)$

   (ii) If the particle strikes a fixed horizontal plane below the sphere at a point distant $\frac{2\sqrt{5}a}{3}$ from the vertical through the centre, show that the depth of the plane below the lowest point of the sphere is $\frac{5a}{48}$.

4. A smooth narrow tube is in the form of a circle of centre O and redius a, which in fixed in a vertical. The tube contains two particles. A of mass $4m$ and B of mass m which are connected by a light inextensible string. Initially A and B are on the same horizontal level as O and the system is released from rest. If, after time t, the line AOB has turned through an angle .

   (i) Show that $5a\left(\frac{d\theta}{dt}\right)^2 = 6g \sin\theta$

   (ii) Find the reaction between B and the tube in terms of m, g and θ.

   (iii) Find a $\frac{d^2\theta}{dt^2}$ in terms of g and θ.

   (iv) Hence find the tension in the string.

Competency Levels relevant to the Competency Level Numbers

Competency levels relevant to the competency level numbers in the Teacher 's Instructional Manual.

Term 1

Combined Maths I

11.3 Solves inequalities including modulus functions.

27.1 Derives the equation of a straight line.

27.2 Derives the equation of a straight line passing through the point of intersection of two given straight lines.

27.3 Positions of two points relative to a given straight line.

27.4 Finds the angle between two straight lines.

27.5 Derives results related to a straight line in terms of the distance of the perpendicular drawn to it from a given point.

28.1 Finds the cartesian equation of a circle.

28.2 Describes the position of a point relative to a circle.

28.3 Describes the position of a straight line relative to a circle.

28.4 Interprets the tangents drawn to a circle from an external point and the chord of contact.

28.5 Interprets the equation $S + \lambda U = 0$.

28.6 Interprets the position of two circles.

28.7 Interprets the equation $S + \lambda S' = 0$.

29 Interprets the conic section.

Combines Maths II

3.10 Interprets mechanical energy.

3.11 Solves problems interpreting the applicability of power appropriately.

3.12 Interprets the effect of an impulsive action.

3.13 Uses Newton's law of restitution to interpret direct elastic impact.

3.14 Investigates the relevant principles to apply them effectively to the motion on a horizontal circle.

3.15 Considers initial velocity as a factor affecting the behaviour of vertical circular motion.

Term 2

Combined Maths I

25.1 Deduces integration results in terms of the ideas about the anti-derivative of a function.

25.2 Uses the theorems on integration to solve problems.

25.3 Reviews the basic properties of a definite integral using the fundamental theorem of calculus.

25.4 Integrates rational functions using appropriate methods.

25.5 Integrates trigonometric expressions on reducing them to standard forms using trigonometric identities.

25.6 Uses the method of changing the variable for integration.

25.7 Solves problems using integration by parts.

25.8 Determines the area of a region bounded by curves using integration.

8.1 Uses various methods for counting.

8.2 Uses of permutations as a technique of solving mathematical problems.

8.3 Uses of combinations as a technique of solving mathematical problems.

21.1 Describes basic series.

21.2 Interprets arithmetico-geometric series.

21.3 Sums series with positive integral powers product terms.

21.4 Sums series using various methods.

Combined Maths II

2.11 Applies various techniques to determine the centre of mass of symmetrical uniform bodies using definition.

2.12 Finds the centre of mass of simple geometrical bodies using definition and integration.

2.13 Finds the centre of mass (centre of gravity) of composite bodies and remaining bodies assuming that the centre of mass and centre of gravity coincide.

2.14 Determines the stability of bodies in equilibrium.

4.1 Interprets the events of a random experiment.

4.2 Applies probability models to solve problems on random events.

4.3 Applies the concept of conditional probability to determine the probability of a random event under given conditions.

4.4 Uses the probability model to determine the independence of two or more events.

4.5 Applies Bayes' Theorem.

Term 3

CombinedMaths I

10.1 Explores the basic properties of the Binomial Expansion.

10.2 Reviews the relation between the terms and coefficients in the Binomial Expansion.

14.1 Extends the number system.

14.2 Interprets complex numbers algebraically.

14.3 Interprets addition geometrically using the Argand diagram.

14.4 Interprets product and quotient geometrically using the Argand diagram.

14.5 Interprets the complex equation of the locus of a variable point.

12.1 Describes basic theories related to matrices.

12.2 Explains special cases of square matrices.

12.3 Describes the transpose and the inverse of a matrix.

12.4 Uses matrices to solve simultaneous equations.

13.1 Interprets the properties of a determinant.

Combines Maths II

3.16 Analyses Simple Harmonic Motion.

3.17 Describes the Nature of a simple Harmonic Motion taking place on a horizontal plane.

3.18 Explains the nature of a Simple Harmonic Motion taking place on a verticle line.

5.1 Introduces the nature of statistics.

5.2 Manipulates data to obtain information.

5.3 Classifies data and information.

5.4 Tabulates data and information.

5.5 Denotes data and information graphically.

5.6 Describes the mean as a measure of central tendency.

5.7 Interprets a frequency distribution using measures of relative positions.

5.8 Uses suitable measures of central tendency to make decisions on frequency distributions.

5.9 Interprets the dispersion of a distribution using measures of dispersion.

5.10 Determines the shape of a distribution by using measures of skewness.

Refferences

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  Stanley Thrones (Publishers) Ltd.- 1993

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  Inequalities
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