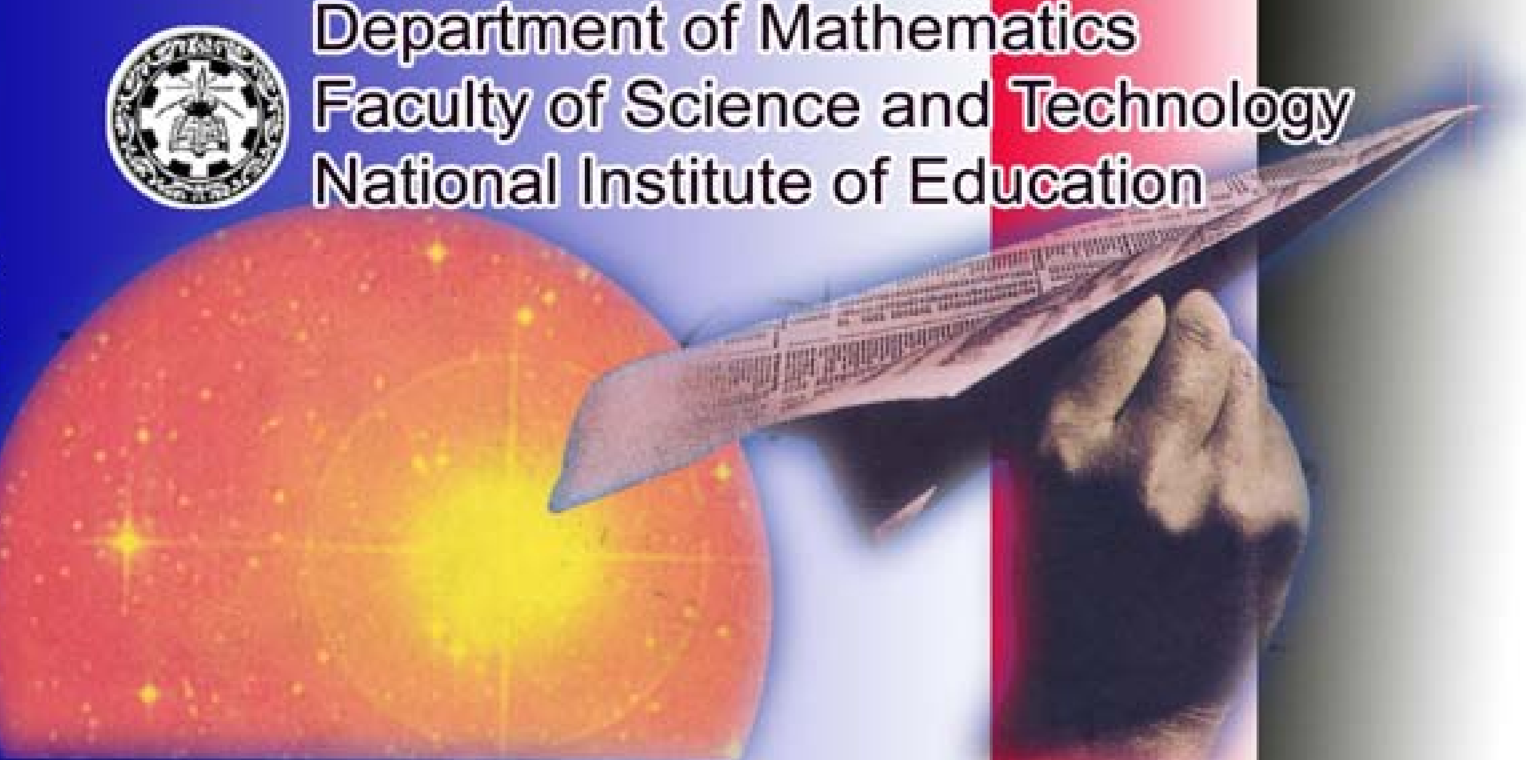
G. C. E. (Advanced Level)

# Combined Mathematics

## Teacher's Instruction Manual

## Grade 12

Department of Mathematics
Faculty of Science and Technology
National Institute of Education



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## Grade 12

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(Implementation in 2009)

Department of Mathematics
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**Teacher's Instructional Manual**

**Grade 12 – 2009**



**ISBN**

**Department of Mathematics**
**Faculty of Science & Technology**
**National Institute of Education**

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## Director General's Message

Curriculum developers of the NIE were able to introduce Competency Based Learning and Teaching curricula for grades 6 and 10 in 2007 and were also able to extend it to 7, 8 and 11 progressively every year and even to GCE (A/L) classes in 2009. In the same manner syllabi and Teacher's Instructional Manuals for grades 12 and 13 for different subjects with competencies and competency levels that should be developed in students are presented descriptively. Information given on each subject will immensely help the teachers to prepare for the Learning – Teaching situations.

I would like to mention that curriculum developers have followed a different approach when preparing Teacher's Instructional Manuals for Advanced Level subjects when compared to the approaches they followed in preparing Junior Secondary and Senior Secondary curricula. (Grades 10, 11)

In grades 6, 7, 8, 9, 10 and 11 teachers were oriented to a given format as to how they should handle the subject matter in the Learning – Teaching process, but in designing AL syllabi and Teacher's Instructional Manuals freedom is given to the teachers to work as they wish.

At this level we expect teachers to use a suitable learning method from the suggested learning methods given in the Teacher's Instructional Manuals to develop competencies and competency levels relevant to each lesson or lesson unit.

Whatever the learning approach the teacher uses it should be done effectively and satisfactorily to realize the expected competencies and competency levels.

I would like to note that the decision to give this freedom is taken, considering the importance of GCE (A/L) examinations and the sensitivity of other stakeholders who are in the education system to the Advanced Level examination. I hope that this Teacher's Instructional Manual would be of great help to teachers.

I hope the information, methods and instructions given in this Teacher's Instructional Manual will provide proper guidance to teachers to awaken the minds of our students.

**Professor Lal Perera**
Director General
National Institute of Education

# Forward

Action taken over long years of the past to retain the known and learn the predetermined has made us little able today to construct even what is. The first curriculum reform of the new millennium on secondary education that comes to being with a drastic change in the learning-teaching process at school level attempts to overcome this inability while bringing about a set of worthy citizens for the country who are capable of revising the known, exploring the undetermined and constructing what might be.

If you are a teacher teaching this subject or any other subject in grades 6 to 11, it will not be difficult for you to align yourself with the new learning-teaching approaches that are recommended in a considerable way for the GCE (A/L) as well. This reform calls the teacher to identify competency levels under each competency and plan activities to achieve them. The teachers entering the new role of transformation should understand that the procedures which emphasize the teacher in the learning-teaching process are of limited use for the present and that it is more meaningful for the children to learn co-operatively sharing their experiences. This situation, however, requires the teachers to provide a new direction for their teaching by selecting new learning–teaching methods that emphasize the student over the teacher.

If you study the Teachers' Instructional Guides (TIGs) prepared by the National Institute of Education for Mathematics, Science, Health & Physical Education, Technology and Commerce subject of grades 6 to 11, you certainly will be able to acquire a good understanding on the student-centred, competency based and activity- oriented approaches we have recommended for learning and teaching. The activities presented in these Guides attempt to bring learning, teaching assessment and evaluation on to the same platform and to help you to adopt co-operative learning techniques on the basis of the 5E Model.

Considering the need to establish an innovative teaching force we have selected just a few activities from the relevant activity continuum incorporated in the TIGs. Yet we have given you a vast freedom to plan your own activities to suit the subject and the class requirements by studying the exemplar activities in the Guides and improving your understanding on the principles underlying the reform. The activities incorporated in the TIG, provide you with four types of information. At the beginning of each activity you come across the final outcome that the children are expected to achieve through each activity. This learning outcome named as 'Competency' is broad and long-term. The competency level stated next highlight one out of the number of abilities that the children have to develop to realize the competency.

The above explanation shows us that the competency levels are more specific and of a shorter duration when compared to the competency. The next section of the Guide presents a list of behaviours that the teacher has to observe at the end of each activity. To facilitate the task of both the teacher and the students, an attempt has been made to limit the number of such behaviours to five. These behaviours referred to as learning outcomes are more specific than the competency level. They include three abilities derived from the subject and two others derived from the learning teaching process. Out of the three subject abilities listed in an order of difficulty, the teacher has to direct the children to realize at least the first two through the exploration. The next section of the activity presents what the teacher should do to engage the children for the exploration. Although the implementation of each and every activity starts with this step of engagement, the teachers should not forget that activity planning should begin with the exploration which is the second 'E' of the 5E Model.

Instructions for the group exploration from the next section of the exemplar activities the teacher plans these instructions in such a way to allow different groups studying different facets of the same problem to reach the expected ends through a variety of learning-teaching methods. For this, further the teacher can select either Inquiry-based Learning carried out through a series of questions or Experiential Learning where children learn by doing. It is the responsibility of the GCE (A/L) teacher to use the knowledge that the children acquire by any of the above methods to solve problems that are specific to the subject or that runs across a number of subjects of the curriculum is meaningful to plan such problem-based learning-teaching methods on the basis of real-life situations. For this you can select dilemmas, hypothetical situations, analogies or primary sources. Some techniques that can be used for the explorations are reading, information management, reflection, observation, discussion, formulation and testing of hypotheses, testing predictions, preparing questions and answers, simulation, problem solving and aesthetic activities such as drawing or composing. There is room here even for memorization although it is considered as a form of mechanical learning.

The students explore in small groups. Instead of depending on the knowledge available to the teacher, they attempt to construct their own knowledge and meaning with the support of the teacher. Moreover, they interact with others in the group to learn from others and also to improve the quality of their exploration findings. All this works successfully only if the teacher is capable of providing the students with the reading material and the other inputs they are in need of. The teacher also has to support student learning throughout the learning process by moving from one group to another. Although it is the discovery that is prominent in this type of learning you have to recognize this as a guided discovery rather than a free discovery. There is no doubt that students learning likewise with instructional scaffolding both by the teacher and the peers acquire a whole lot of worthwhile experiences that they find useful later in life.

Explanation follows the second stage of exploration. The small groups get ready to make innovative, team presentations on their findings. The special feature here is that the children have selected novel methods for their presentations. The responsibility for the presentation is also shared by all members of the group. In the next step of elaboration the children get the opportunity to clarify the unclear, correct the incorrect and fill any gaps that are left. They also can go beyond the known to present new ideas. All activities end with a brief lecture made by the teacher. This stage allows the teacher to go back to the transmission role. The teacher also has to deliver this lecture covering all the important points that the syllabus has prescribed for the relevant competency level. Step 3 of each Activity Plan guides the teachers in this compulsory final elaboration.

To overcome many problems that are associated with the general system of education today, the National Institute of Education has taken steps to move the teachers to the new transformation role recommended for them. This role that starts with a transaction gets extended to a lengthy exploration, a series of student explorations and elaborations and a summative transmission by the teacher. The students involve themselves in the exploration using reading material and other quality inputs provided to them by the teacher.

The students attend school daily to learn joyfully. They achieve a number of competencies that they need to be successful in life and the world of work. They prepare themselves for nation building by developing thinking skills, social skills and personal skills. For the success of all this, an examination system that inquires into the ability of students to face real challenges of life is very much needed in place of an examination system that focuses on the knowledge acquired by children through memorizing answers to model questions.

A number of activities have already begun at the national level to protect the real nature of school-based assessments. The written tests have been minimised to gain recognition for school-based assessments. Compulsory question has been incorporated in the term tests along with a scheme of authentic evaluation to ensure real outcomes of learning. It is the co-ordinated responsibility of all citizens of the country to open up doors for a new Sri Lanka by thriving for the success of this new programme on the basis of sound instructional leadership and quality assurance by the management.

**Deshamanya Dr. (Mrs) I. L. Ginige**
Assistant Director General (Curriculum Development)
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## Message from Commissioner General of Educational Publication

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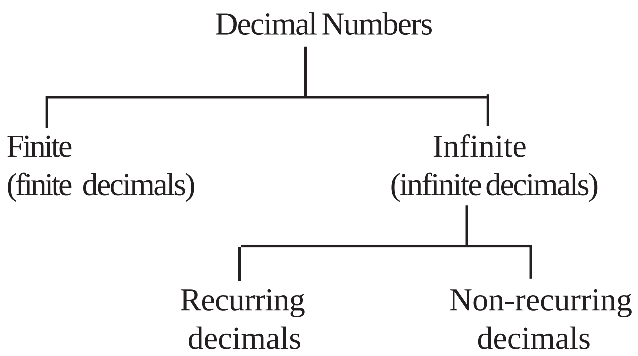
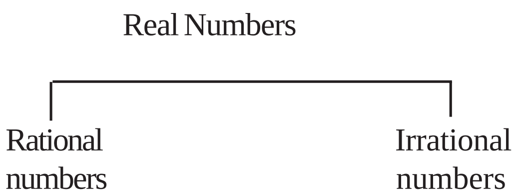
# Content

## Combined Maths I

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
|---|---|---|---|
| 1.1 | 1. Explains the evolution of the number system. | Explain briefly the evolution from the inception of the use of numbers upto the real number system.<br><br>Remind the knowledge of pupils about the sets of natural numbers, integers, rational numbers, irrational numbers and real numbers.<br><br>Show that all sets above are Sub sets of $\mathbb{R}$ and direct the pupils to denote it in a Venn Diagram.<br><br>Introduce the symbols . | 02 |
| | 2. Represents a real number geometrically. | Remind the representation of a real number on a number line. | |
| 1.2 | 1. Classifies decimal numbers |  | 02 |
| | 2. Classifies real numbers |  | |
| | 3. Rationalises the denominator of expressions with surds. | Introduce surds and explain the rationalisation of their denominators. | |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
|---|---|---|---|
| | 5. Deduces the length of a line segment joining two points with given coordinates. | Deduce that if and $B \equiv (x_2,\ y_2)$ then $\lvert AB \rvert = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ . <br> $\lvert AB \rvert$ is always positive. | |
| 26.2 | Deduces the coordinates of the point that divides a line segment joining two given points internally or externally in a given ratio. | The coordinates of a point dividing the line segment AB where and $B \equiv (x_2,\ y_2)$ in the ratio *m*:*n*. <br> (i) internally is given by <br> $P \equiv \left( \frac{nx_1 + mx_2}{m+n},\ \frac{ny_1 + my_2}{m+n} \right)$ <br> (ii) externally is given by <br> , | 03 |
| 26.3 | Finds the area of a triangle when the coordinates of the vertices are given. | The area of the triangle ABC when , $B \equiv (x_2,\ y_2)$ and $C \equiv (x_3,\ y_3)$. Show that the area of a plane figure enclosed by straight lines can be determined by seperating it into triangles. <br> $\triangle = \frac{1}{2} \lvert x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \rvert$ | 01 |
| 16.1 | 1. Explains trigonometric ratios. <br> 2. Introduces trigonometric ratios as circular functions. <br> 3. Introduces the domain and the range of circular functions. | Define trigonometric ratios using the cartesian coordinate system. <br> Show that trigonometric ratio of a variable angle is a function of that angle. Introduce these ratios as circular functions. <br> Introduce the domain and the range of circular functions. | 03 |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
|---|---|---|---|
| | | | |
| | 3. Describes the periodic property of circular functions | When an angle is increased by an integral multiple of 2 , the radius vector reaches the initial position after a single or more revolutions. Therefore and have the same trigonometric ratios. | |
| | 4. Obtains the trigonometric ratios of $\frac{\pi}{2} \pm \theta,\ \pi \pm \theta,\ (-\theta) \ldots$ in terms of the trigonometric ratios of . | Obtain the trigonometric ratios of ... in terms of the trigonometric ratios of using geometrical methods. | |
| | 5. Writes trigonometric ratios of angles of given magnitude. | Direct students to find out the values of sin, cos and tan of the angles , ... | |
| 16.3 | 1. Represents the circular functions graphically. | Introduce the graphs of sin, cos, tan, cot, sec and cosec. | 05 |
| | 2. Draws graphs of combined circular functions. | Direct students to draw the graphs of , , , , , , etc. | |
| | 3. Obtains the general solutions of trigonometric functions. | Obtain the general solutions of;<br>(i) as<br>(ii) $\cos\theta = \cos\alpha$ as<br>(iii) as<br>Where and is value which satisfies the above equations. | |

The quadrant chart printed in the first row of the Guidelines column:

| | |
|---|---|
| (2)<br>sine(+) | (1)<br>all(+) |
| (3)<br>tangent(+) | (4)<br>cosine(+) |

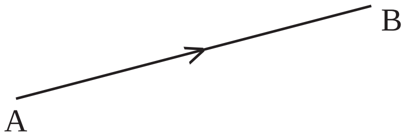
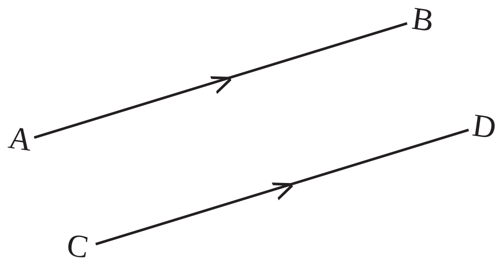
| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
|---|---|---|---|
| 4.1 | 1. Defines a polynomial in a single variable. | Defining a polynomial introduce the terms degree, principal term and principal coefficient of a polynomial. | 01 |
| | 2. Distinguishes among Linear, Quadratic and Cubic functions. | Introduce the general form of a linear function as where , quadratic function as ; Cubic function as , | |
| 4.2 | 3. States the properties of identical polynomials. | Show that if then for all a, $P(a) \equiv Q(a)$ and coefficient of corresponding terms are equal. | 02 |
| | 1. Explains the basic Mathematical operations on polynomials. | Revise the prior knowledge relating to addition, substraction and multiplication. | |
| 4.3 | 2. Devides a polynomial by another polynomial. | Introduce the notation $\frac{P(x)}{Q(x)}$ for divided by . Using examples explain the synthetic division and long division. | 05 |
| | 1. States the algorithm for division. | Explain<br>Dividend = Quotient Divisor + Remainder | |
| | 2. States the remainder theorem. | Express that when a polynomial is divided by the remainder is . | |
| | 3. Proves the remainder theorem. | Prove the remainder theorem. | |
| | 4. States the Factor theorem | Express that if is a factor of | |

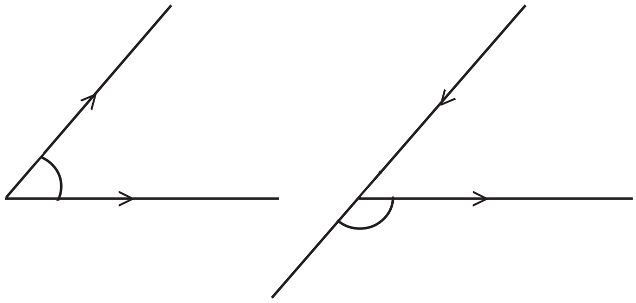
| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
|---|---|---|---|
| | | (iv)<br>(v) $\tan\left(A+B\right)=\dfrac{\tan A+\tan B}{1-\tan A\cdot\tan B}$<br>(vi) $\tan\left(A-B\right)=\dfrac{\tan A-\tan B}{1+\tan A\cdot\tan B}$<br>$\sin C+\sin D=2\sin\left(\dfrac{C+D}{2}\right)\cos\left(\dfrac{C-D}{2}\right)$ | |
| 17.3 | Constructs multiplication formulae | | 03 |
| 17.4 | 1. Derives trigonometric formula for double, trible and half angles. | $=2\cos^2 A-1$<br>$\tan 2A=\dfrac{2\tan A}{1-\tan^2 A}$<br>$\sin 3A=3\sin A-4\sin^3 A$<br>$\cos 3A=4\cos^3 A-3\cos A$<br>Obtain formulae $\sin\dfrac{A}{2}$, and using the above identities. Direct students to prove trigonometric identities related to angles of a triangle. Also show that an expression of the form can be transformed to the form using the above results. | 03 |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
|---|---|---|---|
| 6.1 | 1. Uses index laws. | Remind<br>(i) (ii) $a^m \div a^n = a^{m-n}$<br>(iii) $\left(a^m\right)^n = a^{mn}$ (iv) $\left(ab\right)^m = a^m \times b^m$<br>Where, $a$, and $m$, , as index laws and direct students to derive,<br>, $a^0 = 1 \; ; \; a \neq 0$ ,<br>**root of a real number *a***<br>There are two roots if and $n$ is even. They are equal in magnitude and opposite in sign.<br>, if exists<br>, if<br>, if and $n$ is odd<br>, if and $n$ is even<br>Explain that when and $n$ is odd there is only one root and that it is negative.<br>Substantiate with examples.<br>Give examples to explain that | 04 |
| | 2. Uses logarithmic laws | Using index laws, define logarithms as<br>$\left(a > 0,\ \mathrm{N} > 0\right)$<br>Logarithmic laws,<br>$\log_a\left(\frac{\mathrm{M}}{\mathrm{N}}\right) = \log_a \mathrm{M} - \log_a \mathrm{N}$ | |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
|---|---|---|---|
| | | 6. , <br><br> 7. , $n \in \mathbb{N}$ <br> When <br><br> 8. When *f* is a polynomial function for all <br> . <br><br> 9. If $f(x)=g(x)$ for all values of *x* except at $x=a$ in an interval including *a*, then <br> . <br><br> The proofs of the above theorems are not expected. Explain their use in solving problems with examples. | |
| 22.3 | 1. Expresses $\lim\limits_{x\to a}\dfrac{x^n-a^n}{x-a}=na^{n-1}$ where "*n*" is any rational number. | Prove the theorem for positive integral values of *n*, and deduce it for negative integral values of *n*. <br><br> State that the theorem is true for any rational number *n*. | 03 |
| | 2. Solves problems using above results involving limits. | Direct students to solve suitable problems. | |
| 22.4 | 1. States the sandwich theorem. <br><br> (squeezes lemma) | In an open interval containing a if for all values of *x* including or excluding a $f(x)\le h(x)\le g(x)$ and $\lim\limits_{x\to a} f(x)=l=\lim\limits_{x\to a} g(x)$ then $\lim\limits_{x\to a} h(x)=l$. <br> The proof of this theorem is not necessary. | 03 |
| | 2. States that $\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1$ | State that (*x* is measured in radians) | |
| | 3. Proves the above result. | When *x* is measured in radians proves that <br> by a geometrical method. Using the <br> above results deduce that | |

## Combined Maths II

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
|---|---|---|---|
| 1.1 | 1. Explains the differnece between scalar quantities and scalars. | Explain that a quantity having only a magnitude and expressed using a certain measuring unit is called a scalar quantity and that numerical value without unit is a scalar. | 03 |
| | 2. Defines a vector quantity. | Explain vector quantities as those with magnitude, direction and obeying triangle law of addition. [Triangle law of addition will be given later]. | |
| | 3. Represents a vector geometrically. | Present that the vector represented by the line segment AB from A to B is denoted by . <br> | |
| | 4. Defines a "geometrical vector" | Explain that a line segment with a magnitude and a direction is geometrically a vector.<br>A vector does not have dimensions although a vector quantity has dimensions. | |
| | 5. Expresses the algebraic notation of a vector quantity. | Show that the vector " " is denoted by the symbol or (In print, letters in bold print are used to denote vectors). State also that different letters are used to denote different vectors. | |
| | 6. Classifies vectors. | Show how vectors can be classified into free vectors, sliding vectors and tied vectors. | |
| | 7. Defines the modulus of a vector. | Introducing the magnitude of a vector as its modulus, show that the modulus of is denoted by . Explain that is always a positive scalar, because it is the length of a line segment. | |
| | 8. States the conditions necessary for two vectors to be equal. | Vectors with equal magnitude and in the same direction are called equal vectors.<br> | |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
|---|---|---|---|
| | 15. Multiplies a vector by a scalar. | When is a vector and *k* is a scalar, introduce that as *k* times . Discuss the cases when and . Give examples. Describe the vector .<br> | |
| | 16. Identifies the angle between two vectors. | Introduce the angle between two vectors as the angle, , between their directions. | |
| | 17. Identifies parallel vectors. | State that vectors whose lines of action are parallel are called "parallel vectors". | |
| | 18. State the conditions necessary for two vectors to be parallel. | Show that and are parallel, if where *k* is a non zero scalar.<br>If and are two non zero vectors and and are scalars the sufficient and necessary conditions for is . | |
| | 19. Defines a "unit vector". | Define a vector of unit magnitude as a unit vector. Show that if is a given vector in the direction of .<br>then $\underline{u} = \frac{\underline{a}}{\lvert\underline{a}\rvert}$ | |
| | 20. Resolves a vector in any two given directions. | Show how a given vector can be resolved in two given directions by constructing a parallelogram with given vector as a diagonal and adjacent sides in the required directions.<br>Show how a given vector can be resolved in two given mutually perpedicular directions by constructing a rectangle with the given vector as a diagonal. | |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
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| | | which a right handed screw descends when it is screwed from the line of action of towards the line of action of , is a unit vector in a direction perpendicular to both and . { forms a right handed set} State that this is also known as the cross product of and . If or or , define as null vector. | |
| | 2. States that the scalar product of two vectors is a scalar. | Explain that is a scalar. Show that $\underline{a}\cdot\underline{b}=0$ if and also that , where | |
| | 3. States the properties of scalar product. | (i) Commutative law<br>(ii) Distributive law | |
| | 4. Interprets scalar product geometrically. | Show that, when is a unit vector, is the orthogonal prjectors of on . | |
| | 5. Solves simple geometric problems involving scalar product. | Explain with examples. | |
| 2.1 | 1. Describes the concept of a particle. | State that a particle is considered as a solid body having very small dimensions compared with other distances related to its motion.<br>Show that as a particle can be considered as a sphere of zero radius having a mass, it can be represented geometrically by a point. | 04 |
| | 2. Describes the concept of a force. | Introduce force as an action which creates a motion in a body at rest or which changes the nature of the motion in the case of a moving body. | |
| | 3. States that a force is a localized vector. | A force has a magnitude, direction and a line of action. Therefore it can be treated as a localized vector. | |
| | 4. Represents a force geometrically. | State that Newton (N) is the unit by which the magnitude of a force is measured.Show that a force can be represnted by a line segment whose length is proportional to the madnitude of the force and drawn in its direction. | |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
|---|---|---|---|
| | 10. Resolves a given force into two components in two given directions. | Show the method of resolving a given force in two directions by constructing a parallelogram with its adjacent sides in the two given directions and its diagonal representing the given force.<br>Show that the effect of the two resolved parts is same as the effect of the given force. | |
| | 11. Resolves a given force into two components perpendicular to each other. | State that a force is resolved into two components perpendicular to each other for the convenience in solving problems and obtain the two components. | |
| 2.2 | 1. Determines graphically the resultant of three or more coplanar forces acting at a point. | Present the graphical method (polygon method) of determining the resultant of three or more coplanar forces acting at a point. | 06 |
| | 2. Determines the resultant of three or more coplanar forces acting at a point by resolution. | Show that, if X and Y are the algebraic sums of the components of the forces when resolved in two given perpendicular directions then their resultant is also given by and the angle $\alpha$ which the resultant makes with the direction of X is given by .<br>Direct students to solve problems using these results.<br>Condition to be fulfilled for the equilibrium of a coplanar system of forces acting on a particle. | |
| | 3. States the conditions necessary for a system of coplanar forces acting on a particle to be in equilibrium. | The algebraic sums of the resolved components in two given perpendicular directions are zero. i.e. X=0, Y=0. | |
| 2.3 | 1. Explains what is meant by equilibrium. | Show that a particle acted upon by a system of coplanar forces is in equilibrium, if the resultant of the system of forces is zero. | 06 |
| | 2. States the conditions for equilibrium of a particle under the action of two forces. | Show that if a particle under the action of two forces P and Q, is in equilibrium then P and Q are equal in magnitude and opposite in direction (i.e. they are collinear). | |

# Combined Maths

## Grade 12

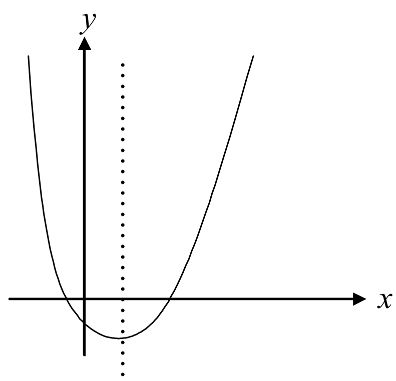
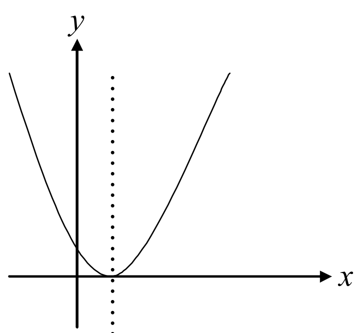
## Teacher's Instruction Manual

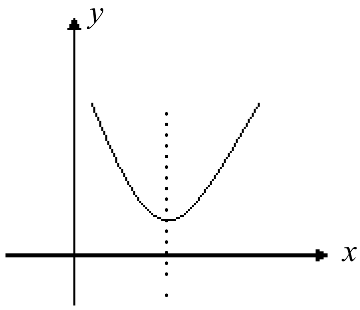
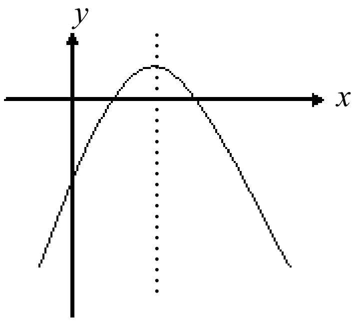
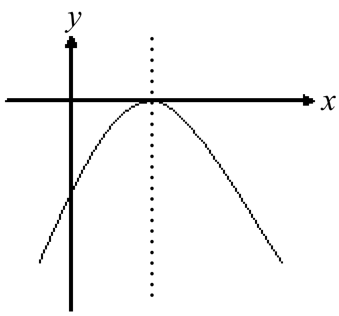
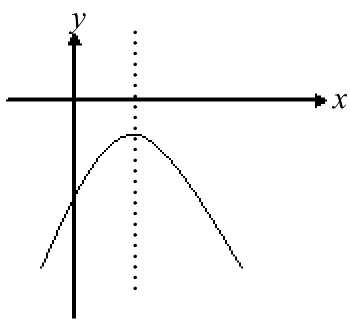
## Second Term

## Combined Maths I

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
|---|---|---|---|
| 11.1 | 1. Defines inequalities | When "a" and "b" are real numbers<br>(i) $a > b$ if and only if $(a\text{-}b)$ is positive and<br>(ii) $a < b$ if and only if $(a\text{-}b)$ is negative<br><br>Explain the concepts in these definitions using the number line | 04 |
| | 2. Represents inequalities on a real number line. | Explain inequalities in terms of the number line. | |
| | 3. Denotes inequalities in terms of interval notation. | Introduce the following special sets which are intervals in $\mathbb{R}$ .<br>When $a,\ b \in \mathbb{R},\ \ a < b,$<br>Interval          Notation<br>$\{x \in \mathbb{R}\ \ a \le x \le b\}$     $[a,\ b]$<br>$\{x \in \mathbb{R}\ \ a \le x < b\}$     $[a,\ b)$<br>$\{x \in \mathbb{R}\ \ a < x \le b\}$     $(a,\ b]$<br>$\{x \in \mathbb{R}\ \ a < x < b\}$     $(a,\ b)$<br>Explain following intervals as well.<br>$\{x \in \mathbb{R}\ \ x \ge a\}$     $[a,\ +\infty)$<br>$\{x \in \mathbb{R}\ \ x > a\}$     $(a,\ +\infty)$<br>$\{x \in \mathbb{R}\ \ x \le a\}$     $(-\infty,\ a]$<br>$\{x \in \mathbb{R}\ \ x < a\}$     $(-\infty,\ a)$ | |
| | 4. States the Trichotomy law. | When $x$ and $y$ are any two numbers either one of the following is true.<br>$x > y$ , $x < y$ , $x = y$ | |
| | 5. States and proves fundamental results on inequalities. | Results.<br>When $a,\ b,\ c \in \mathbb{R}$<br>(i) $a > b$ and $b > c \Rightarrow a > c$<br>(ii) $a > b \Rightarrow a + c > b + c$<br>(iii) $a > b$ and $c > 0 \Rightarrow ac > bc$<br>(iv) $a > b > 0$ and $c < 0 \Rightarrow ac < bc$<br>(v) $a > b$ and $c = 0 \Rightarrow ac = bc = 0$ | |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
|---|---|---|---|
| | | (vi) $a > b$ and $c > d \Rightarrow a + c > b + d$<br>(vii) $a > b > 0$ and $c > d > 0 \Rightarrow ac > bd$<br>(viii) $a > b > 0 \Rightarrow \frac{1}{a} < \frac{1}{b}$<br>(ix) $a < b < 0 \Rightarrow \frac{1}{a} > \frac{1}{b}$<br>(x) For $a > b > 0$ and $n$ is a positive rational number, $a^n > b^n$ and $a^{-n} < b^{-n}$ | |
| 11.2 | 1. Solves inequalities involving polynomials. | When $f(x)$ and $g(x)$ are two polynomials of $x$ inequalities such as,<br>$f(x) \geq g(x)$ , $f(x) > g(x)$<br>$f(x) \leq g(x)$ , $f(x) < g(x)$<br>Linear functions, Quadratic functions are considered. Direct students to find the limit of values of $x$ satisfying inequalities including linear, quadratic functions. Present the solutions using the inequality signs and set notation. | 04 |
| | 2. Solves inequalities including rational functions. | Consider the rational functions, with the order less than or equal to two, in the dinominator and the numerator. | |
| 3.1 | 1. Introduces linear functions. | Introduce $f(x) = ax + b,\ a,\ b \in \mathbb{R},\ a \neq 0$ as a linear function. | 15 |
| | 2. Explains what a quadratic function is. | Recall that the function $f(x) = ax^2 + bx + c$ is called a quadratic function when $a \neq 0$ and $a,\ b,\ c \in \mathbb{R}$. | |
| | 3. Explains the properties of a quadratic function. | Showing that the quadratic function can be written in the form $a(x+p)^2 + q,\ p,\ q \in \mathbb{R}$.<br>Discuss the sign of the quadratic function for various values of $x$ in terms of the above form.<br>Show that the graph of the function is symmetrical about the line $x = -p$.<br>Discuss the behaviour of the quadratic function, | |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
|---|---|---|---|
| | | (i) When $\Delta > 0$, $a > 0$ and $a < 0$<br>(ii) When $\Delta = 0$, $a > 0$ and $a < 0$<br>(iii) When $\Delta < 0$, $a > 0$ and $a < 0$<br>Here $\Delta = b^2 - 4ac$ is called the discriminant of the function $f(x) = ax^2 + bx + c$.<br>Show that $q$ is the maximum or the minimum value of the function, accordingly as $a$ is positive or negative.<br>Explain with examples existence or non-existence of real zeros. | |
| | 4. Draws the graph of a quadratic function. | Direct students to draw the graphs of quadratic functions for the cases when $b^2 - 4ac > 0$, $b^2 - 4ac < 0$ and $b^2 - 4ac = 0$.<br>Where $a > 0$, $a < 0$. | |
| | 5. Describes the different types of the graph of the quadratic function. | Emphasises the properties of the quadratic function by means of the graphs drawn by students.<br>$b^2 - 4ac > 0$<br>$a > 0$<br><br>$b^2 - 4ac = 0$<br>$a > 0$<br> | |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
|---|---|---|---|
| | | $b^2 - 4ac < 0$<br>$a > 0$<br><br>$b^2 - 4ac > 0$<br>$a < 0$<br><br>$b^2 - 4ac = 0$<br>$a < 0$<br><br>$b^2 - 4ac < 0$<br>$a < 0$<br> | |
| 3.2 | 1. Introduces what a quadratic equation is. | State that when $a \neq 0$, $a$, $b$, $c \in \mathbb{R}$, $ax^2 + bx + c = 0$ which gives the zero points of the quadratic function is called the quadratic equation. | 15 |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
|---|---|---|---|
| | 2. Finds the roots of a quardatic equation. | Prove that a quardatic equation cannot have more than two distinct roots.<br>Prove that a quardatic equation in a single variable can have only two roots generally.<br>If roots are and show that<br>$\alpha = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$ , $\beta = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ | |
| | 3. Describes the nature of the roots of a quardatic equation. | Accordingly as $b^2 - 4ac > 0$, $b^2 - 4ac = 0$ and $b^2 - 4ac < 0$,<br>show that the roots of the quardatic equation are real and distinct or real and coincident or imaginary.<br>Show the converse is also true.<br>Explain that the necessary and sufficient condition for the roots of the quardatic equation to be real is $b^2 - 4ac \geq 0$,<br>$\Delta = b^2 - 4ac$ is called the discriminant of the equation $ax^2 + bx + c = 0$. | |
| | 4. Expresses the sum and product of the roots of quadratic equation in terms of its coefficient. | If the roots of the equation $ax^2 + bx + c = 0$ are $\alpha$ and $\beta$ show that $\alpha + \beta = \dfrac{-b}{a}$ and $\alpha\beta = \dfrac{c}{a}$. | |
| | 5. Construct quadratic equations whose roots are symmetric functions of and $\beta$. | If the roots of the equation $ax^2 + bx + c = 0$ are $\alpha$ and $\beta$ obtain equations of whose roots are symmetric functions of and $\beta$. | |
| | 6. Solves problems involving quadratic functions and quadratic equations. | Direct students to solve suitable problems. | |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
|---|---|---|---|
| 7 | 1. Defines the exponential function $\left(e^x\right)$. | State that the sum of the infinite series, $1+\frac{x}{\lfloor 1}+\frac{x^2}{\lfloor 2}+\frac{x^3}{\lfloor 3}+\ldots+\frac{x^n}{\lfloor n}+\ldots$ is denoted by $e^x$ and that it is known as the exponential function. | 03 |
| | 2. Expresses the domain and the range of the exponential function. | Here, since $x$ is an exponent, $e^x$ is reffered as the exponential function.<br>State if<br>$f\left(x\right)=e^x$, then $D_f=\mathbb{R}$, $R_f=\mathbb{R}^+$ | |
| | 3. States that $e$ is an irrational number. | $e$ is the sum of the above infinite series when $x=1$ and is a positive irrational number. | |
| | 4. Estimates the value of $e$. | $f\left(1\right)=e=1+\frac{1}{\lfloor 1}+\frac{1}{\lfloor 2}+\frac{1}{\lfloor 3}+\ldots+\frac{1}{\lfloor n}+\ldots=2.718$<br>Stress that $e$ is a positive irrational number. | |
| | 5. Describes the properties of the $e^x$. | State that<br>(i) $e^0=1$<br>(ii) $e^{(x_1+x_2)}=e^{x_1}\cdot e^{x_2}$<br>(iii) $e^{(x_1-x_2)}=\frac{e^{x_1}}{e^{x_2}}$<br>(iv) For rational values of $r$, $\left(e^x\right)^r=e^{rx}$<br>(v) $\lim\limits_{x\to\infty} e^x=\infty$<br>(vi) $\lim\limits_{x\to-\infty} e^x=0$ | |
| | 6. States that the exponential function too satisfies the laws of indices. | Deduce that $e^x$ satisfies the laws of indices using the properties (i) and (ii) above. | |
| | 7. Draws the graph of $y=e^x$ | Present the graph of $y=e^x$. At this stage, the presentation of its shape is sufficient. | |
| | 8. Draws the graph of $y=e^{-x}$ | Guide the students to draw the graph of $y=e^{-x}$. | |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
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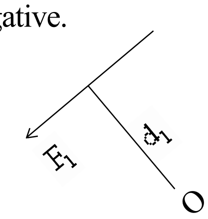
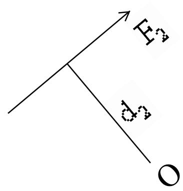
| | 9. Defines the natural logarithmic function. | Explain that $\ln x$ defined as $y = \ln x \Leftrightarrow x = e^y$, $x \in \mathbb{R}^+$ is kown as the natural logarithmic function. | |
| | 10. Expresses the domain and the range of the logarithmic function. | If $g(x) = \ln x$<br>then $D_g = \mathbb{R}^+$, $R_g = \mathbb{R}$ | |
| | 11. Expresses the properties of $\ln x$. | (i) $\ln(xy) = \ln x + \ln y$<br>(ii) $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$<br>(iii) $\ln(x^p) = p\ln x$; $x > 0,\ y > 0$ | |
| | 12. Draws the graph of $y = \ln x$. | Present the graph of $y = \ln x$.<br>At this stage the shape of the graph alone is sufficient. | |
| | 13. Defines the function $a^x$ for $a > 0$. | The Function $a^x$ is defined as $a^x = e^{x\ln a}$. | |
| | 14. Expresses the domain and the range of $y = a^x$. | If $h(x) = a^x$<br>$D_h = \mathbb{R}$<br>$R_h = \mathbb{R}^+$ | |
| 19 | Solves trigonometric equations | (i) Equations of the form $\sin\theta = \sin\alpha$, $\cos\theta = \cos\alpha$, $\tan\theta = \tan\alpha$<br>(ii) Equations that can be factorized.<br>(iii) Equations that can be solved using Pythagorean identities, Addition formulae and Multiplication formulae.<br>(iv) Equations that can be solved by assuming the formulae of double angles, trible angles or half angles.<br>Solutions of equations that can be converted to the above forms are also expected.<br>Also emphasize that equations of the form $a\cos\theta + b\sin\theta = c$ can also be solved. | 04 |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
|---|---|---|---|
| 20 | 1. Defines inverse trigonometric functions. | (i) Explain that if $y = \sin x$, the value of $x$ when $y$ is given is stated as $x = \sin^{-1} y$ and that $x = \sin^{-1} y$ is not a function. But it can be made a function by limiting the domain of $y = \sin x$. Here, the domain of sin $x$ generally limited as $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. The value of $x$ for sin $x$ are denoted as $x = \sin^{-1} y$. By interchanging $x$ and $y$ it can be denoted by $y = \sin^{-1} x$ Then $-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$.<br>(ii) Explian similarly that $y = \cos^{-1} x$ is defined such that $0 \le \cos^{-1} x < \frac{\pi}{2}$ and that $\tan^{-1} x$ is defined such that $-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$.<br>(iii) Define, also, $y = \tan^{-1} x$ and explain that the values included in the domain $-\frac{\pi}{2} < x < \frac{\pi}{2}$ are its principal values. | 06 |
| | 2. States the domain and the range of inverse trigonometric functions. | $y = \sin^{-1} x$; Domain $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, Range $[-1, 1]$.<br>$y = \cos^{-1} x$; Domain $[0, \pi]$, Range $[-1, 1]$.<br>$y = \tan^{-1} x$; Domain $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, Range $(-\infty, \infty)$. | |
| | 3. Draws the graphs of inverse trigonometric functions. | Draw the graphs of the functions $y = \sec^{-1} x$, $y = \mathrm{cosec}^{-1} x$ and $y = \cot^{-1} x$. State their domain and the range.<br><br>Draw the graphs of these functions. | |
| | 4. States simple relationships between inverse trigonometric functions. | Direct students to solve problems involving inverse trogonometric functions. | |

## Combined Maths II

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
|---|---|---|---|
| 2.4 | 1. Describes a rigid body. | Describe a rigid body as one in which the distance between any two points remains unchanged when it is subjected to external forces of any magnitude. | 04 |
| | 2. States the principle of transmission of forces. | Explain that a force acting on a rigid body can be considered to act at any point on its line of action. | |
| | 3. Explains the translation and rotation of a force. | Show that a linear motion as well as a rotation can be created by a force. | |
| | 4. Defines the moment of a force about a point. | Present the definition that the moment of the force about a point is the product of the magnitude of the force and the perpendicular distance to its line of action from the given point. | |
| | 5. Explains the physical meaning of moment. | Build up the concept of moment as a measure of tendency to rotate about a certain point as a result of an external force acting on a rigid body.<br>(For two dimensions only).<br>Make them release that what is measured by moment is a turning effect about a line perpendicular to the plane determined by that point and the line of action of the force | |
| | 6. Finds the magnitude of the moment about a point and its sense. | Demonstrate that the sense of the moment can be considered as clockwise or anticlockwise. Explain that according to sign convention an anticlockwise moment is considered positive and a clockwise moment is considered negative.<br><br>Moment of about $= F_1 \times d_1$.<br><br>Moment of $F_2$ about $= F_2 \times d_2 = -F_2 d_2$ | |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
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| | 7. States the dimensions and units of moments. | Show that dimension is  and the unit is NM. | |
| | 8. Represents the magnitude of the moment of a force about a point geometrically. | Explain that the magnitude of the moment of a force F denoted in magnitude, direction and position by $\overrightarrow{AB}$ about a point is twice the area of the triangle OAB. | |
| | 9. Determines the algebraic sum of the moments of the forces about a point in the plane of a coplanar system of forces. | Direct students to solve problems of finding the algebraic sum of the moments of a system of coplanar forces about a point of the plane. | |
| | 10. States the general principle of moment of a system of forces. | State that the algebraic sum of moments of a system of coplanar forces about a point in the plane is equal to the moment of the resultant of the system about the same point. (Proof is not expected). Explain using suitable examples. | |
| 2.5 | 1. Finds the resultant of two forces acting on a rigid body. | • When the two forces are **not parallel**<br>As the two forces meet at a point show that their resultant can be determined by applying the law of parallelogram of forces.<br><br>• When the two forces are **parallel**<br>Introduce that forces acting on lines parallel to each other are known as parallel forces.<br><br>When two parallel forces are acting in the same direction they are said to be like forces, while those acting in opposite directions are said to be unlike forces.<br><br>Explain that the parallelogram law of forces cannot be used to find the resultant of two parallel forces.<br><br>If the two forces are P and Q and their resultant is R, and if the lines of action of P, Q and R intersect a certain straight line at A, B and C respectively.<br><br>$R = P + Q$, if P and Q are like<br>$R = P - Q$, $(P > Q)$ if P and Q are unlike. | 04 |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
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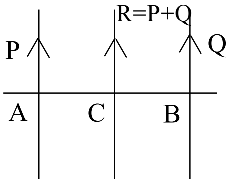
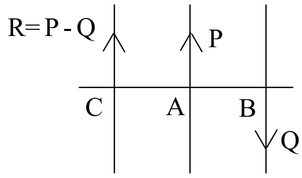
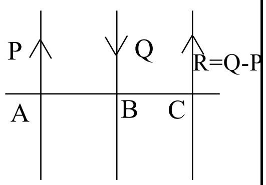
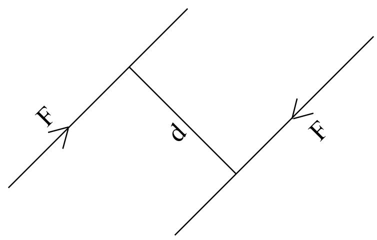
| | |  <br>$P \cdot AC = Q \cdot CB$ When P<Q  | |
| | 2. States the conditions necessary for the equilibrium of two forces acting on a rigid body. | Show that for two forces acting on a rigid body are in equilibrium , the two forces should be collinear, equal in magnitude and opposite in direction. | |
| | 3. Describes a couple. | Introduce a couple as a pair of parallel unlike (opposite) forces of equal magnitude. In this case, show that the vector sum of the two forces is zero, the algebraic sum of the moments of the two forces about any point is not zero. Therefore there is no translation, but only a rotation of the body. | |
| | 4. Calculates the moment of a couple | Show that the moment of a couple about any point in the plane of the couple is the product of the magnitude of one of the forces forming the couple and the perpendicular distance between the lines of action of the two forces.<br>Mention that according to sign convention the anticlockwise (left handed) moments are considered as positive whereas clockwise (right handed) moments are considered to be negative.<br><br>Moment of the couple $= F \times d$<br>$= -F \times d$ | |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
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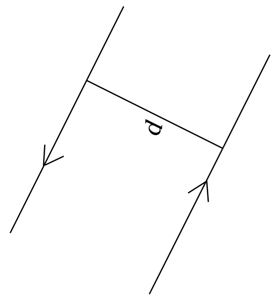
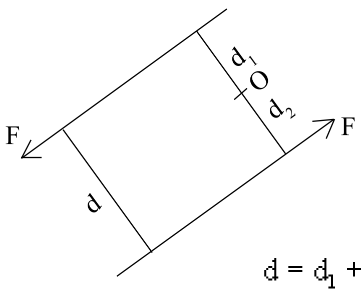
| | |  Moment of the couple $= F \times d$ | |
| | 5. States that the moment of a couple is indipendent of the point about which the moment of the forces is taken. |  $d = d_1 + d_2$<br>The moment of two forces about<br>$= Fd_1 + Fd_2 = F(d_1 + d_2)$<br>$= F \times d$<br>Show that the same result will be obtained by taking moments about any point in the plane of the couple. | |
| | 6. States the necessary conditions for two coplanar couples to be equivalent | As two coplanar couples with moments of equal magnitude and same sense produce same rotation, they are equivalent. | |
| | 7. States the conditions for two coplanar couples to balance each other. | As the resulting rotation when two coplanar couples of equal magnitude but of opposite sense, produce zero rotation, two such couples are said to balance each other. | |
| | 8. Combines coplanar couples | Explain that the algebraic sum of moment should be taken in finding the moment of the combination of two or more coplanar couples. | |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
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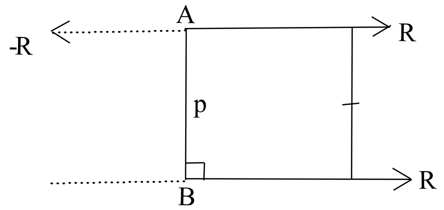
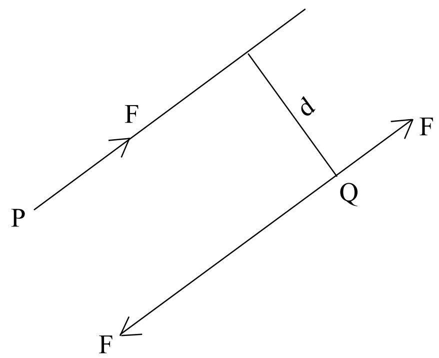
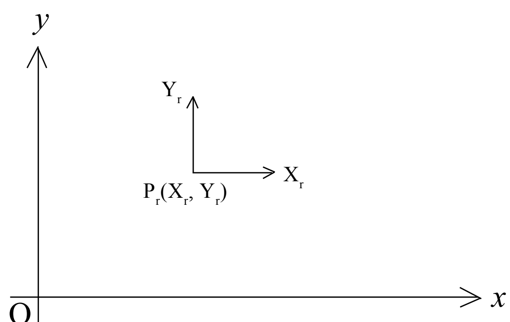
| 2.6 | 1. Reduces a couple and a single force acting in its plane into a single force. | <br>$G = pR \Rightarrow p = \frac{G}{R}$<br>Show that a couple of moment G and a single force R acting in its plane is equivalent to a force equal and parallel to a force R acting at a distane $\frac{G}{R}$ from the point of action of it. | 10 |
| | 2. Shows that a force acting at a point is equivalent to the combination of an equal force acting at another point together with a couple. | Show that the force F acting at P is equivalent to the combination of a force F acting at a point Q and a couple of moment $G = F \times d$, where $d$ distance between the lines of action of the two forces F.<br> | |
| | 3. Reduce a system of coplanar forces to a single force acting at an arbitrary point O and a couple G. | Show how a system of coplanar force is reduced to a single force R acting at origin O and a couple of moment G.<br><br>r = 1, 2, 3, ........, n | |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
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| | | Consider the system of coplanar forces F, acting at $P_r(X_r, Y_r)$.<br>Where r = 1, 2, ......, n<br>$\underline{R} = \sum_{r=1}^{n} \underline{F}_r = \sum_{r=1}^{n}(X_r\underline{i} + Y_r\underline{j})$<br>$= \left(\sum_{r=1}^{n} X_r\right)\underline{i} + \left(\sum_{r=1}^{n} Y_r\right)\underline{j}$<br>$= X\underline{i} + Y\underline{j}$<br>Where $X = \sum_{r=1}^{n} X_r$ and<br>$Y = \sum_{r=1}^{n} Y_r$.<br>Magnitude of R<br>$R = \sqrt{X^2 + Y^2}$<br>If R makes an angle $\theta$ with the $x$ - axis<br>$\theta = \tan^{-1}\frac{Y}{X}$ and<br>$G = \sum_{r=1}^{n}(x_r Y_r - y_r X_r)$<br>anti clockwise. | |
| | 4. Reduces any coplanar system of forces to a single force and a couple acting at any point in that plane. | Show how it can be reduced to a single force $R'$ acting at the point P($x$, $y$) in the plane and a couple $G'$ obtain that,<br>$R' = R$ and<br>$G' = G - xY + yX = 0$<br>Present the following necessary and sufficient conditions for two coplanar systems of forces to be equivalent.<br>That when each system of forces is resolved seperately along two axes Ox and Oy perpendicular to each other selected in the plane of the two systems of forces each algebraic sum of components of one system of forces is equal to the corresponding algebraic sum of the other system of forces. | |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
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| | | $X' = X$, $Y' = Y$, where X and Y are the algebraic sums of one system of forces along O$x$ and O$y$ and $X'$, $Y'$ are the algebraic sums of the other system of forces along the same axes.<br>If the algebraic sum of the moments of the two systems of forces about any point $(h, k)$ in the plane, are $G_1'$ and $G_2'$, $G_1' = G_2'$. | |
| | 5. (i) Reduction of a system of coplanar forces to a single force when it reduces to a single force. | When a system of coplanar forces is reduced to a single force $\underline{R} = (X\underline{i} + Y\underline{j})$ acting at the origin O and couple of moment G, discuss cases,<br>(i) $\underline{R} \neq \underline{0}$ (i.e. at least one of X or Y should not be zero). Show that when G = 0 it is reduced to a single force through another point. | |
| | (ii) Reduction of a system of forces to a couple. | (ii) Show that $\underline{R} = \underline{0}$ (i.e., X = 0 and Y = 0) and $G \neq 0 \Rightarrow$ couple of moments. | |
| | (iii) Expresses conditions necessary for equilibrium. | (iii) If $\underline{R} = 0$ (i.e., X = 0 and Y = 0) and G = 0.<br>Show that the systems of forces are in equilibruim. Give suitable examples for each condition. | |
| | 6. Finds the magnitude, direction and the line of action of the resultant of a coplanar system of forces. | Magnitude of the resultant $\lvert\underline{R}\rvert = \sqrt{X^2 + Y^2}$.<br>If $\theta$ is the angle made by R with the $x$-axis,<br>$\theta = \tan^{-1}\left(\frac{Y}{X}\right)$.<br>Show that the equation of the line of action is<br>$G - xY + yX = 0$. | |
| 2.7 | 1. States conditions for the equilibrium of three coplanar forces acting on a rigid body. | (i) If three co-planar forces acting on a rigid body are in equilibrium then their lines of action pass through the same point or else they are parallel.<br>Explain that this is a necessary condition only. | 10 |

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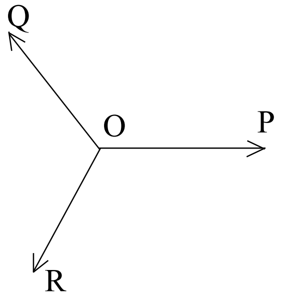
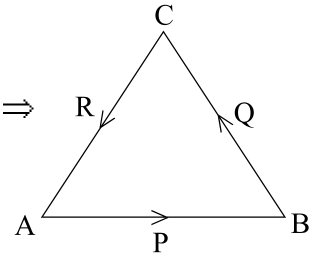
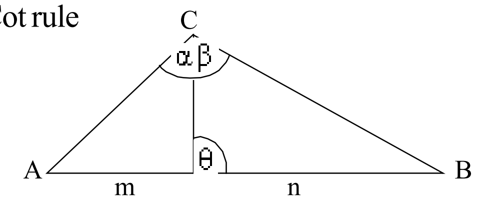
| | | (ii) State again the law of triangle of forces and its converse.<br>(This has been presented under the equilibrium of a particle.)<br><br>Three forces P, Q, R in equilibrium acting at O<br><br>Corresponding triangle of forces, ABC.<br>Using the triangle of forces - ABC, show that,<br>$\frac{P}{AB} = \frac{Q}{BC} = \frac{R}{CA}$<br>use this result in solving problems.<br><br>(iii) Lami's theorem (Presented under equilibrium of a particle)<br>Explain that this theorem also can be applied in solving problems involving equilibrium of three coplanar non parallel forces whose line of action are concurrent.<br>(iv) The algebraic sums of resolutes along two perpendicular directions seperately are zero.<br>(v) Cot rule<br><br>$n \cot A - m \cot B = (m+n) \cot \theta$ or<br>$m \cot \alpha - n \cot \beta = (m+n) \cot \theta$<br>Show that, using examples the above results can be used in solving problems.<br>In solving problems make use of the geometrical properties of the diagram as far as possible. | |

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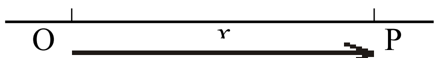
| 3.1 | 1. Defines "distance". | Distance: When a variable point P moves from a point A to a point B present the length measured along the path form A to B as the distance travelled by the point. | 10 |
| | 2. States dimensions and standard units of distance. | Dimensions of distance : L<br>Standard unit is m.<br>Introduce other units of distance such as mm, cm, km. | |
| | 3. Defines speed. | Speed:- Speed is defined as the rate of change of distance with respect to time. | |
| | 4. States dimensions and standard units of speed. | Dimension of speed $LT^{-1}$;<br>Standard unit is $m s^{-1}$.<br>Introduce other units of speed such as $km h^{-1}$. | |
| | 5. States that distance and speed are scalar quantities. | Distance is a quantity with a magnitude measured with some unit. Time is also same. Since they do not possess a direction demonstrate that distance and speed are scalar quantities. | |
| | 6. Defines average speed. | If the total distance travelled from A to B is *s,* and the time taken is *t* define that $\frac{s}{t}$ as the average speed during the time interval *t*. | |
| | 7. Defines instantaneous speed. | The speed of a moving particle at a given instant of time is defined as the instantaneous speed at that time. | |
| | 8. Defines uniform speed. | If the instantaneous speed of a particle over a certain time interval remains constant, define that speed as a uniform speed. | |
| | 9. Defines position coordinates of a particle undergoing rectilinear motion. | <br>The position coordinate of a point P moving along a line '*l*' is denoted by '*x*', and defined as $x = \pm\lvert OP\rvert$ depending whether its position is on the right or left of the point O.<br>Explain that *x* is a function of time *t*. | |

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| | 10. Defines Displacement. | Define displacement as the change in the position coordinate during a given time interval.<br>If the position coordinate of the particle P at times $t_1$ and $t_2$ are $x_1$ and $x_2$ respectively, show that the displacement *s* of the particle during the time interval $(t_1,\ t_2)$ is defined as $s = (x_2 - x_1)$.<br>Explain that the displacement is a vector quantity.<br>Show also that accordingly as $s > 0$ or $s < 0$ its direction is taken as $\rightleftarrows$. | |
| | 11. Expresses the dimension and standard units of displacement. | Dimension L; Standard unit is *m* (meter)<br>Explain other units too in terms of the standard units. | |
| | 12. Defines velocity. | Velocity: Velocity is defined as the rate of change of displacement with respect to time. | |
| | 13. Expresses dimension and units of velocity. | Dimension of velocity *v* is $\mathrm{LT}^{-1}$ and standard unit is $ms^{-1}$.<br>Also explain other units in terms of the standard unit. | |
| | 14. Defines average velocity. | If during the time interval $(t_1,\ t_2)$ the displacement $s = (x_2 - x_1)$.<br>Average velocity too is a vector quantity. Average velocity during that time interval is defined as $\dfrac{x_2 - x_1}{t_2 - t_1}$.<br>The average velocity of a particle during a small interval $[t, t+h]$ is given by $= \dfrac{x(t+h) - x(t)}{h}$. | |
| | 15. Defines instantaneous velocity. | Introduce this velocity as the instantaneous velocity of the particle at time '*t*'. Show that $v = \dfrac{ds}{dt}$ and explain that velocity is also a function of time. | |

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| | | If the displacement '*s*' is measured from a fixed point O then show that *v* can also be written as $v = \frac{ds}{dt}$, i.e., velocity is the rate of change of displacement with respect to time. By convention, accordingly as $v > 0$ or $v < 0$, its direction is taken as $\rightleftarrows$. | |
| | 16. Defines uniform velocity. | If the instantaneous velocity of some particle at any moment in some time interval is constant, then define the velocity of the particle in that time interval to be uniform. | |
| | 17. Draws the displacement time graph | Using suitable examples provide an understanding of how displacement time graph is drawn. | |
| | 18. Finds the average velocity between two positions using the displacement time graph. | If $s_1$ and $s_2$ are the displacements corresponding to the times $t_1$ and $t_2$ show that the average velocity $\frac{s_2 - s_1}{t_2 - t_1}$ can be obtained by the gradient of the line $P_1P_2$. Here $P_1$ and $P_2$ are points on the displacement- time curve corresponding to times $t_1$ and $t_2$ respectively. Show that the gradient of the tangent drawn to the displacement time graph, at a particular point gives the instantaneous velocity at that moment. Obtain the relation, instantaneous velocity $= \frac{ds}{dt}$ $=$ gradient | |
| | 19. Determines the instantaneous velocity between two positions using the displacement time graph. | Show that the gradient of the tangent drawn to the displacement time graph, at a particular point gives the instantaneous velocity at that moment. Obtain the relation instantaneous velocity $= \frac{ds}{dt}$ $=$ gradient | |
| | 20. Defines accelaration | Acceleration: Define acceleration of the displacement-time curve at time *t* of the tangent as the rate of change of velocity with respect to time. | |

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| | 21. Expresses the dimensions and units of acceleration. | Express that the dimension of acceleration is $\mathrm{LT}^{-2}$ and the Standard unit is, meters per second $\left(ms^{-2}\right)$. Mention other units of acceleration. | |
| | 22. Defines average acceleration. | If the velocities of a particle are $v_1$ and $v_2$ at times $t_1$ and $t_2$ respectively, then define the average acceleration of the particle during the time interval $[t_1, t_2]$ as $\frac{v_2 - v_1}{t_2 - t_1}$. Show that average acceleration $\gtreqless 0$ accordingly as $v_2 \gtreqless v_1$ | |
| | 23. Defines instantaneous acceleration. | The average acceleration during a small time interval $[t, t+h]$ is $\frac{v(t+h)-v(t)}{h}$.<br>As $h \to 0$, the limit of the average acceleration $=$ $\lim_{h\to 0}\frac{v(t+h)-v(t)}{h} = a$.<br>Introduce *a* as the instantaneous acceleration at time *t*.<br>Show that $a = \frac{dv}{dt}$. This is known as the acceleration of the particle at time *t*.<br>Explain that acceleration is a function of time.<br>Emphasize the fact that acceleration is the rate of change of velocity with respect to time. | |
| | 24. Defines uniform acceleration. | If the acceleration is constant in a certain time interval define that motion as a motion with uniform acceleration. | |
| | 25. Defines retardation. | State that when the acceleration is negative it is called a retardation. | |
| | 26. Draws the velocity time graph. | Provide an understanding of drawing velocity time graphs using suitable examples. | |

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| | 27. Finds average accelaration using the velocity time graph. | If the velocities corresponding to times $t_1$ and $t_2$ are $v_1$ and $v_2$ respectively, then show that the average acceleration during the time interval $[t_1, t_2] = \frac{v_2 - v_1}{t_2 - t_1}$. It can be obtained as the gradient of the line $P_1P_2$. Here $P_1$ and $P_2$ are two points corresponding to times $t_1$ and $t_2$ in the velocity time curve. | |
| | 28. Finds the acceleration at a given instant using velocity - time graph. | Show that the gradient of the tangent drawn at a given point to the velocity time graph gives the instantaneous acceleration at the time represented by the point. Deduce that instantaneous acceleration $a = \frac{dv}{dt}$ (gradient). Show also that $a = v\frac{dv}{ds}$. | |
| | 29. Finds displacement using velocity time graph. | Explain that displacement during a certain time-interval is given by the area between the graph and the time axis. (an area below the time axis is assigned a negative sign). | |
| | 30. Draws velocity time graphs for different types of motion. | Direct students to draw $v$ - $t$ graphs for<br>1. Position of rest - zero velocity.<br>2. Uniform velocity.<br>3. Uniform acceleration.<br>4. Uniform retardation.<br>5. A combinations of such motions. | |
| | 31. Solves problems using displacement time and velocity-time graph. | Direct students to solve problems related to rectilinear motion with uniform acceleration. | |

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| 3.2 | 1. Derives kinematic equations for a particle moving with uniform acceleration. | Introduce the standard symbols for<br>initial velocity - $u$ ; final velocity - $v$<br>acceleration - $a$ ; time - $t$ and<br>displacement - $s$ and derive the kinamatic equations.<br>$v = u + at$<br>$s = \frac{1}{2}(u + v)t$<br>$s = ut \frac{1}{2}at^2$<br>$v^2 = u^2 + 2\,a\,s$ | 08 |
| | 2. Derives kinematic equations using velocity - time graph. | Guide students to derive kinematic equations using velocity-time graph. | |
| | 3. Uses kinematic equations for vertical motion under gravity. | Show that in this case the acceleration should be replaced by $g$ = (the acceleration due to gravity) which is a constant.<br>Show that $g$ is taken approximately as $10ms^{-2}$ and remind assumptions. | |
| | 4. Uses velocity - time and displacement - time graphs to solve problems related to vertical motion under gravity. | Explain using suitable examples. | |
| 3.3 | 1. Describes the concept of frame of reference for one dimensional motion. | Introduce that a particle moving (on a straight line) and an axis fixed rigidly to the particle so that it lies along the line constitute a frame of reference. | 07 |
| | 2. Describes the motion of one body relative to the other when two bodies are moving on a straight line. | Explain with examples. | |

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| | 3. States the principle of relative displacement for two bodies moving along straight line. | If the displacement of particles P and Q relative to the frame of reference O on a straight line are $s_{P,O}$ and $s_{Q,O}$ respectively express the displacement of Q relative to P as $s_{Q,P} = s_{Q,O} + s_{O,P}$ .<br>Show that $s_{O,P} = -s_{P,O}$ . | |
| | 4. States the principle of relative velocity for two bodies moving along a straight line. | Obtain $v_{Q,P} = v_{Q,O} + v_{O,P}$ by differentiating the displacement equation with respect to time. | |
| | 5. States the principle of relative acceleration for two bodies moving along a straight line. | Obtain the relation $a_{Q,P} = a_{Q,O} + a_{O,P}$ by differentiating the velocity equation with respect to time. | |
| | 6. Finds relative displacement, relative velocity and relative acceleration for two particles moving along two parallel paths. | Explain using suitable examples.<br>Consider only cases where the distance between the parallel paths is negligible. | |
| | 7. Uses kinematic equations and graphs related to motion for two bodies moving along the same straight line with constant relative acceleration. | Guide students to solve related problems. | |

# Combined Maths

## Grade 12

## Teacher's Instruction Manual

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## Third Term

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## Combined Maths I

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| 3.1 | 1. Defines increment and incremental ratio. | An infinitesimal change in a variable is called an increment. An increment change in $x$ is usually denoted by $\Delta x$ . Emphasize that $\Delta x$ is a symbol and not the product of $\Delta$ and $x$.<br><br>Let $y$ be a function of $x$. Then $x$ is called the independent variable and $y$ the dependent variable. Let the increment in $x$ be $\Delta x$ and the corresponding increment in $y$ be $\Delta y$ .<br><br>When $x_0$ is an element in the domain of $y = f(x)$ then $\dfrac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$ is called the increment ratio at $x = x_0$ .<br>When $x_0$ belongs to the range of $x$. | 04 |
| | 2. Discusses the existence of a derivative. | If the above incremental ratio approaches a finite limit as $\Delta x \to 0$, then the function $f$ is said to be differentiable with respect to $x$ at $x = x_0$ . This finite limit is called the derivative of $f$ at $x = x_0$ or the differential coefficient of $f$ with respect to $x$.<br>It is denoted by the symbols $f'(x_0)$ or $\left[\dfrac{d}{dx}(f(x))\right]_{x=x_0}$ or $\left(\dfrac{d}{dx}\right)_{x=x_0}$ .<br><br>$f'(x_0) = \lim\limits_{\Delta x \to 0} \dfrac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$<br><br>Using suitable examples, explain that the derivative of $f(x)$ does not exist at $x = x_0$ in the instances given below.<br><br>(i) When $f$ is not defined in an open interval including $x = x_0$ .<br><br>(ii) When the $\lim\limits_{\Delta x \to 0} \dfrac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$ does not exist. | |

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| | | (iii) When the limit $\lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$ is not finite.<br>Explain these, using examples. | |
| | 3. Defines the derivative function. | Let $f$ be a function of $x$. If the derivative of $f$ exists at some point $x$, the function $f'$ whose domain consists of all such $x$ values is called the derivative function of $x$.<br>That is $(f')(x) = f'(x)$<br>It is denoted by $\frac{df(x)}{dx}$ or $\frac{dy}{dx}$<br>When $y = f(x)$<br>$\therefore f'(x) = \frac{df(x)}{dx} = \frac{dy}{dx}$. | |
| | 4. Represents the derivative geometrically. | Explain that the derivative represent the gradient of the tangent at $\mathrm{P}(x, y)$ to the graph of the function $y = f(x)$. | |
| 23.2 | 1. Differentiates a function from first principles. | When $n$ is a rational number, showing the method of determining the derivative of $x^n$ and the derivatives of basic trigonometric functions from first principles present the proofs of the following results.<br>$\frac{d}{dx}(x^n) = n\,x^{n-1}$ $\quad \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$<br>$\frac{d}{dx}(\sin x) = \cos x$ $\quad \frac{d}{dx}(\sec x) = \sec x \tan x$<br>$\frac{d}{dx}(\cos x) = -\sin x$ $\quad \frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$<br>$\frac{d}{dx}(\tan x) = \sec^2 x$ | 05 |

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| 23.3 | 1. States fundamental results of derivatives. | Prove that<br>(i) When $k$ is a constant and $f(x) = k$ then $f'(x) = 0$.<br>(ii) If $f(x) = kg(x)$ then $f'(x) = kg'(x)$.<br>(iii) If $f(x) = g(x) \pm h(x)$ then $f'(x) = g'(x) \pm h'(x)$ | 05 |
| | 2. Solves problems using fundamental results of derivatives. | Direct students to solve problems using $\frac{d(x^n)}{dx} = nx^{n-1}$ and the above theorems after explaining suitable examples. | |
| | 3. States the results of derivatives of a product, quotient and a composite function. | (i) $\frac{d}{dx}\left[f(x) \cdot g(x)\right] = f(x)\frac{d}{dx}\left[g(x)\right] + g(x)\frac{d}{dx}\left[f(x)\right]$<br>(ii) $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}\left[f(x)\right] - f(x)\frac{d}{dx}\left[g(x)\right]}{\left\{g(x)\right\}^2}$, $g(x) \neq 0$<br>(iii) If $y$ is a function of $u$ and $u$ is a function of $x$ then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ (Chain Rule) and its extension.<br>Present the above results.<br>Proofs are not required. | |

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| | 4. Finds the derivatives of fundamental trigonometric functions. | State that these should be memorized as standard results. Direct students to solve a variety of problems using these results. | |
| 24.1 | 1. Obtains the derivatives of inverse circular functions. | Deduces that<br>(i) $\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}, \quad -1 < x < 1$<br>(ii) $\frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1-x^2}}, \quad -1 < x < 1$<br>(iii) $\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}, \quad -\infty < x < \infty$ | 03 |
| | 2. Solves problems using the derivatives of inverse circular functions. | Differentiates various functions using the above formula. | |
| | 3. Writes the derivative of the exponential function. | State that $\frac{de^x}{dx} = e^x$ | |
| | 4. Solves problems using the derivative of the exponential function. | Direct students to solve relevant problems. | |
| | 5. Deduces the derivative of $(\ln x)$ | Deduce that $\frac{d(\ln x)}{dx} = \frac{1}{x}$, $x > 0$ derivative of $(\ln x)$ | |
| | 6. Deduces the derivative of $a^x$. | Deduces that $\frac{d\left(a^x\right)}{dx} = (\ln a)a^x$. | |
| | 7. Solves problems using the derivatives of $\ln x$ and $a^x$. | Present examples and give exercises. | |

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| 23.5 | 1. Defines implict functions. | Presenting various equations of the form $F(x, y) = 0$ in variables $x$ and $y$ show that if $y$ be made the subject there may be more than one answer for $y$ in terms of $x$, if a suitable domain is selected, in some cases, $y$ is a function of $x$. These functions determined by an equation of the form $F(x, y) = 0$ are called implicit functions. | 03 |
| | 2. Determines the derivatives of implicit functions. | Show separately the derivatives of each implict function obtained from an equation in $x$ and $y$ of the form $F(x, y) = 0$. Then show that all the above derivatives can be obtained at once, by differentiating the original equation with respect to $x$. | |
| | 3. Differentiates parametric functions. | When $x$ and $y$ are differentiable functions of the parameter "$t$" <br> $$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}; \quad \frac{dx}{dt} \neq 0$$ <br> Then $\frac{dy}{dx}$ is a function of parameter $t$. <br> Show that <br> $$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}; \quad \frac{dx}{dt} \neq 0$$ | |
| 23.6 | 1. Determines derivatives of higher order. | When $y$ is a function of $x$ then $n^{th}$ order differential of $y$ is obtained by differentiating $y$ with respect to $x$, $n$ times. This is denoted by $\frac{d^n y}{dx^n}$ or $f^{(n)}(x)$ or $y^{(n)}$ | 03 |
| | 2. Differentiates functions of various types. | Direct the students to solve various types of problems using the above results. | |

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| 24.1 | 1. Defines average rate and instantaneous rate. | When $y = f(x)$, $y_1 = f(x_1)$ and $y_2 = f(x_2)$, $\frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$ is called the average rate of change in $y'$ with respect to *x* when the variable *x* is changing from $x_1$ to $x_2$.<br>If $x_2 = x_1 + \Delta x$, then the instantaneous rate of change in *y* with respect to *x* at the instant $x = x_1$ is defined as $f'(x_1) = \lim_{\Delta x \to 0} \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}$. | 03 |
| | 2. States that velocity and acceleration are the derivatives, with respect to time, of displacement and velocity respectively. | The displacement *s* of moving particle is a function of the time *t*. The instantaneous rate of change, $\frac{ds}{dt}$, of *s* with respect of the time *t* is defined as the velocity (*v*) of the particle at *t*. The velocity *v* of a moving particle is also a function of the time *t*. The instantaneous rate of change, $\frac{dv}{dt}$, of *v* with respect to the time *t* is defined as the acceleration (*a*) of the particle. | |
| | 3. Differentiates between increasing and decreasing functions. | If for all $x_1, x_2 \in [a,b]$ when $x_1 < x_2$ if $f(x_1) < f(x_2)$ then *f* is said to be an increasing function in $[a,b]$.<br>If for all $x_1, x_2 \in [a,b]$ when $x_1 < x_2$ if $f(x_1) \le f(x_2)$ then *f* is said to be a non decreasing function in $[a,b]$.<br>If for all $x_1, x_2 \in [a,b]$ when $x_1 < x_2$ if $f(x_1) > f(x_2)$ then *f* is said to be a decreasing function in $[a,b]$.<br>If for all $x_1, x_2 \in [a,b]$ when $x_1 < x_2$ if $f(x_1) \ge f(x_2)$ then *f* is said to be a non increasing function in $[a,b]$. | |

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| | 4. Determines the behaviour of a function in an interval using derivatives. | Explain the determination of increasing or decreasing functions within an interval using the derivatives of the function. State that the condition necessary for the function Y = $f(x)$, for any $x \in [a,b]$ is $\frac{dy}{dx} > 0$. Convince by a diagram. State the corresponding condition for decreasing functions too. | |
| 24.2 | Writes down the equations of the tangent and normal at a given point to a curve. | Consider the function *f* defined for an open interval that includes $x_1$. Let *m* denote $\lim_{\Delta x \to 0} \frac{f(x_1+\Delta x)-f(x_1)}{\Delta x}$ if it exists.<br>Then *m* is called the gradient of the tangent of the curve $y = f(x)$ at the point $(x_1, y_1)$.<br>Equation of the tangent is $y - y_1 = m(x - x_1)$.<br>And show that the gradient of the normal is $-\frac{1}{---}$ and the equation of the normal is $y - y_1 = -\frac{1}{m}(x - x_1)$.<br>When $m \neq 0$, $\frac{1}{m}$ is called the gradient of the normal and the equation of the normal therefore,<br>$y - y_1 = \frac{1}{m}(x - x_1)$. | 03 |
| 24.3 | 1. Defines stationary points on a given function. | In a differentiable function defined $x = c$, such that $f'(c) = 0$, is define as a stationary point. Explain with suitable example. | 03 |
| | 2. Describes what a relative maximum and a relative minimum. | If a function *f* defined at $x = c$, is differentiable at $x = c$ and if then the point where $x = c$ defined as a critical point.<br>Show that, if for $x \in (a - \theta,\ a + \theta)$ there exists $\theta > 0$ such that $f(a) \leq f(b)$, then *f* is said to have a relative minimum at $x = a$. | |
| | 3. Employs the first derivative test to find the maximum and minimum points of a function. | Describe the 1st derivative test.<br>Present the first derivative test. | |

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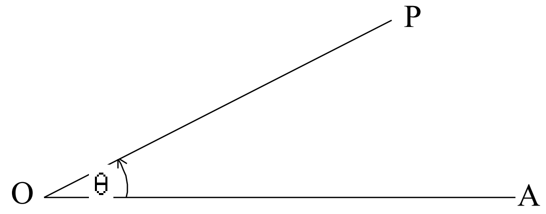
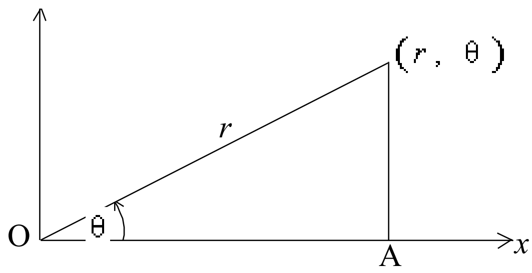
| | 4. States that there exists critical points, which are neither a local maximum nor a local minimum. | Discuss the examples, cases in which, even though $f'(c) = 0$, the point where $x = c$ is neither a relative maximum nor a relative minimum. | |
| | 5. Uses the second differential coefficient to test whether a turning point in a given function is a local maximum or a local minimum | If $f'(a) = 0$ and $f''(a) > 0$, there is a minimum at $x = a$.<br>If $f'(a) = 0$ and $f''(a) < 0$ there is a maximum at $x = a$. | |
| 24.4 | Sketches the graph of function. | Direct the students to draw graphs of function using the above principles.<br>Examples involving horizontal and vertical asymptotes are also included. | 03 |
| 24.5 | Uses derivatives to solve day-to-day problems. | Direct students to solve problems involving maximum and minimum in day-to-day activities. | 05 |
| 9 | 1. Identifies mathematical induction as a method of logical proof. | Introduce Mathematical induction as a method of proving a mathematical result depending on "*n*" positive integers step by step logically.<br>Introduce the following step used in mathematical induction.<br>(i) Show the result is true when $n = 1$.<br>(ii) Assuming that the result is true when $n = p$ (a positive integer) and prove that the result is true when $n = p + 1$<br>(iii) Finally state that the result is true for all positive integers which $n$ can take.. | 05 |
| | 2. Uses the method of mathematical induction to solve problems. | Give the exercises with examples. | |

## Combined Maths II

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
|---|---|---|---|
| 3.4 | 1. Introduces the system of polar coordinates. | <br>If O is a fixed point through O, OA is a fixed line and P is a variable point such that $OP = r$ and $\angle AOP = \theta$, then state that the polar coordinates of P are defined as $(r, \theta)$. Here $r \geq 0$, and the angle $\theta$ measured anticlockwise is taken to be positive. Show that a point can also be denoted uniquely by polar co-ordinates. | 06 |
| | 2. Derives the relation between polar co-ordinates and cartesian coordinates. | <br>With respect to the system of axes O$xy$ if $P \equiv (x, y)$ derive that $x = r\cos\theta$, $y = r\sin\theta$.<br>Then show that $\underline{r} = x\underline{i} + y\underline{j}$<br>$\Rightarrow \underline{r} = r\left(\cos\theta\underline{i} + \sin\theta\underline{j}\right)$ | |
| | 3. Expresses the position of a particle in vectors. | Consider a system of cordinate axes O$xy$ in the plane of the motion of particle P. Take unit vectors along O$x$ and O$y$ as $\underline{i}$ and $\underline{j}$ respectively. Then if $(x, y)$ are the position coordinates of P the position vector of P is given by $\underline{r} = x\underline{i} + y\underline{j}$.<br>Show that $x$ and $y$ are functions of time.<br>Then $\underline{r} = x(t)\underline{i} + y(t)\underline{j}$. | |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
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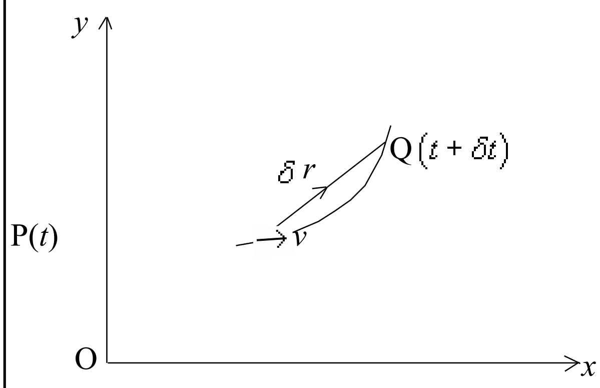
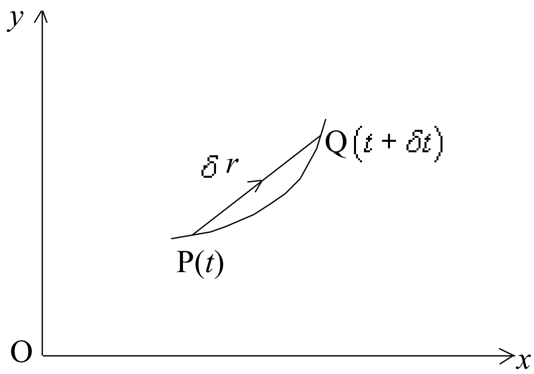
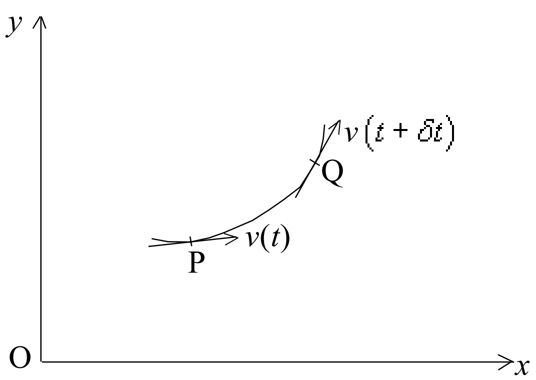
| | |  | |
| | 4. Introduces the average velocity of a particle moving on a plane. | Take the position of a point at time $t$ relative to the origin O as P and its position at time $t + \delta t$ as Q. Here, if $\overrightarrow{OP} = \underline{r}$ . Show that the average velocity in the time interval $\delta t$ can be expressed as $\frac{\overrightarrow{PQ}}{\delta t} = \frac{\delta \hat{\underline{r}}}{\delta t}$ . | |
| | 5. Defines instantaneous velocity. | <br>State that the instantaneous velocity of the particle at time $t$ is defined as $\underline{v} = \lim_{\delta t \to 0} \frac{(\underline{r} + \delta \underline{r}) - \underline{r}}{\delta t} = \lim_{\delta t \to 0} \frac{\delta \underline{r}}{\delta t} = \frac{d\underline{r}}{dt}$ | |
| | 6. Defines the average acceleration of a particle moving on a plane. |  | |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
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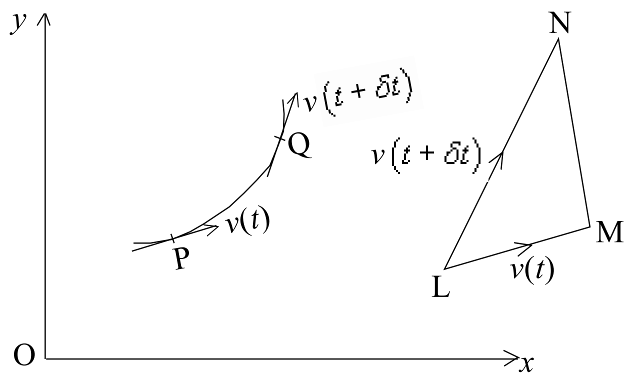
| | | State that the position of the particle at time $t$ is P and the velocity is $\underline{v}(t+\delta t)$.<br>Define the average acceleration of the particle in the time interval $\delta t$ is equal to $\dfrac{\underline{v}(t+\delta t)-\underline{v}(t)}{\delta t}$. | |
| | 7. Defines the instantaneous acceleration of a particle moving on a plane | The instantaneous acceleration of a particle at time $t$ is defined as<br>$\underline{a} = \lim_{\delta t \to 0} \dfrac{\underline{v}(t+\delta t)-\underline{v}(t)}{\delta t}$<br>$= \lim_{\delta t \to 0} \dfrac{\delta \underline{v}}{\delta t} = \dfrac{d\underline{v}}{dt}$<br>**The direction of the acceleration**<br><br>If the velocity $\underline{v}(t)$ at time $t$ is represented by $\overrightarrow{LM}$ and the velocity $\underline{v}(t+\delta t)$ at time $t+\delta t$ by $\overrightarrow{LN}$ (in magnitude and direction), then $\overrightarrow{MN}$ represents $\underline{v}(t+\delta t)-\underline{v}(t)$.<br>Emphasize that as $\lim_{\delta t \to 0} \dfrac{\underline{v}(t+\delta t)-\underline{v}(t)}{\delta t}$ gives the acceleration, $\overrightarrow{MN}$ gives the direction of the acceleration as $\delta t \to 0$, i.e., the acceleration of the particle is directed towards the concave side of its path. | |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
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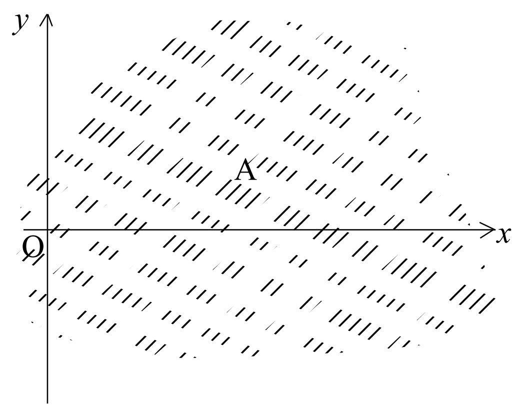
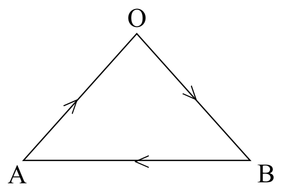
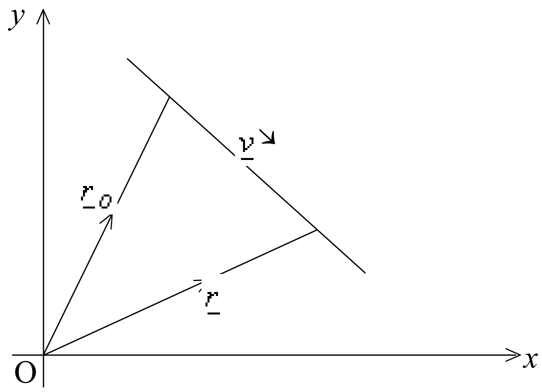
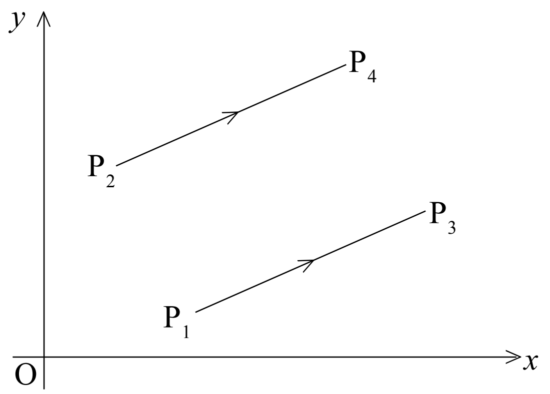
| | 8. Finds velocity $y$ when the position vector is given as a function of time. | The instantaneous velocity<br>$= \underline{v} = \frac{dx}{dt}\underline{i} + \frac{dy}{dt}\underline{j}$<br>$= \left[\dot{x}\underline{i} + \dot{y}\underline{j}\right]$<br>Remind that $\underline{i}$ and $\underline{j}$ are constant unit vectors.<br>Express that, if $\frac{dx}{dt} = \dot{x}$ and $\frac{dy}{dt} = \dot{y}$<br>$\underline{v}$ can be written a $\underline{v} = \dot{x}\underline{i} + \dot{y}\underline{j}$.<br>Show that the magnitude of the instantaneous velocity<br>$= \sqrt{\dot{x}^2 + \dot{y}^2}$ and its direction is given by $\tan\theta = \frac{\dot{y}}{\dot{x}}$<br>where $\theta$ is the angle made by the direction of the velocity with the positive direction of the $x$-axis. Explain also that the magnitude of the velocity is known as its speed and its direction is along the tangent at the point P to the curve of its path. | |
| | 9. Finds the acceleration when its position vector is given as a function of time. | Express that, since $\underline{v} = \dot{x}\underline{i} + \dot{y}\underline{j}$, $\underline{a}$ can be obtained as<br>$\underline{a} = \frac{d\underline{v}}{dt} = \frac{d}{dt}(\dot{x})\underline{i} + \frac{d}{dt}(\dot{y})\underline{j}$<br>$= \ddot{x}\underline{i} + \ddot{y}\underline{j}$ | |
| 3.5 | 1. Describes the concept of frame of reference for two dimensional motion. | Since only two dimensional motion is considered here, it is sufficient to consider a frame of reference as follows.<br> | 06 |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
|---|---|---|---|
| | | Consider a body A moving in a plane. Select Cartesian axes perpendicular to each other in the plane of motion and rigidly fixed to the body A. Introduce that a fixed set of points relative to the axes (can be extended at will) is known as the frame of reference of A. | |
| | 2. Presents vectorially the terms displacement, velocity and acceleration relative to a frame of reference for two dimensional motion. | Remind the earlier definitions of displacement, velocity and acceleration. Explain that if the displacement of a particle relative to the origin of a frame of reference is $\underline{r}$ then the velocity $\underline{v} = \frac{d\underline{r}}{dt}$ and the acceleration $\underline{a} = \frac{d\underline{v}}{dt}$.<br>Explain using suitable examples. | |
| | 3. Describe the (relative) motion of one particle relative to another. | Show that if $\underline{r}_{A,O}$ and $\underline{r}_{B,O}$ are the position vectors of A and B respectively, relative to the origin O then the position vector of B relative to A, $\underline{r}_{B,A}$ is given by $\underline{r}_{B,A} = \underline{r}_{B,O} - \underline{r}_{A,O}$ | |
| | 4. Express the principle of relative displacement. |  <br>$\underline{r}_{B,A} = \underline{r}_{B,O} - \underline{r}_{A,O}$<br>$\underline{r}_{B,A} = \underline{r}_{B,O} + \underline{r}_{O,A}$ | |
| | 5. Expresses the principle of relative velocity. | Derive the velocity of B relative to A, $\underline{v}_{B,A} = \underline{v}_{B,O} + \underline{v}_{O,A}$ by differentiating the displacement equation with respect to time. | |
| | 6. Expresses the principle of relative acceleration. | By differentiating the velocity equation with respect to time, derive that $\underline{a}_{B,A} = \underline{a}_{B,O} + \underline{a}_{O,A}$. | |
| | 7. Describes the velocity of one body relative to another body. | Direct students to find the velocity of one body relative to another body when the relative velocity is uniform. | |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
|---|---|---|---|
| | 8. Determines the path of one body relative to another body using the principle of relative motion. | Direct students to find the path of one body relative to another body when the relative velocity is uniform. | |
| 3.6 | Uses the principle given above to solve problems involving relative motion. | Direct the students to find the following when the relative velocity is uniform.<br>(i) The velocity of a body relative to another.<br>(ii) The shortest distance between the two bodies and time taken to reach it.<br>(iii) The time taken by two bodies to collide the condition for collision and position of collision.<br>(iv) The time taken to complete a given path.<br>(v) Present solutions of problems involving motions relative to water or air. | 10 |
| 3.7 | 1. Finds the vector equation of a line on which a body is moving. | <br>Derive that the vector equation is $\underline{r} = \underline{r}_o + \lambda \underline{v}$. | 06 |
| | 2. Finds the position of two particles at any instant. | <br>Show that the distance between the two particles at any instant when<br>$\overrightarrow{OP_4} = \underline{r}_1 + \underline{v}_1 t$<br>$\overrightarrow{OP_3} = \underline{r}_2 + \underline{v}_2 t$ is given by $\lvert \overrightarrow{P_3P_0} \rvert$ | |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
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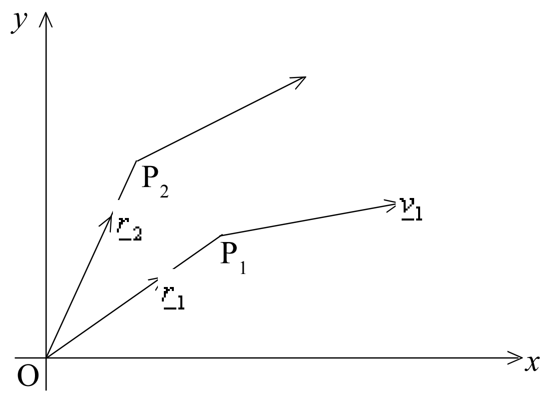
| | 3. Finds the relative position and the relative velocity of a particle using vectors. | <br>Show that displacement $(P_2, P_1) = \underline{r}_1 - \underline{r}_2$ and velocity $(P_2, P_1) = \underline{v}_1 - \underline{v}_2$. | |
| | 4. Finds the shortest distance between two particles and time taken to attain it. | As the distance between the two particles is a function of time direct the students to find the shortest distance and the time taken to attain it using differentiation. | |
| | 5. Finds the condition for impact of two particles, the position vector at the point of impact and the time for impact. | Show that the condition for impacts of the two particles is<br>(i) $\overrightarrow{P_3P_4} = \underline{0}$ or<br>(ii) $vel\ (\overrightarrow{P_2, P_1}) = \overrightarrow{P_2P_1}$<br>Direct the students to find the time for impact and to find the position vector of the particles by using any one of the above conditions. | |
| | 6. Solves problems involving relative motion of three particles. | Direct students to use vector methods to find the true motion of a particle (relative to the Earth) when the motion of the particle relative to two particles are known separately. | |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
|---|---|---|---|
| 3.8 | 1. Introduces a projectile. | Introduce a projectile as a particle or a body moving freely under gravity. | 08 |
| | 2. Describes the terms "velocity of projection" and "angle of projection" | When a particle or a body is projected with a velocity $u$ inclined at an angle $\alpha$ to the horizontal introduce that $u$ is the velocity of projection and $\alpha$ is the angle of projection. | |
| | 3. State that the motion of a projectile can be considered as two motions, separately, in the horizontal and vertical directions. | Explain that the velocity component is constant for the horizontal motion and the acceleration, which is that due to gravity, is constant for the vertical motion. | |
| | 4. Applies the kinematic equations to interpret motion of a projectile. | Show that the following equations can be used with $a = g$ ,<br>Horizontally $s = ut$<br>Vertically $v = u + at$<br>$s = ut + \frac{1}{2}at^2$<br>$v^2 = u^2 + 2a\,s$ | |
| | 5. Calculates the components of velocity of a projectile after a given time. | Derive that the horizontal and vertical components of velocity at a time $t$, are $\dot{x} = u\cos\alpha$ and $\dot{y} = u\sin\alpha - gt$ . | |
| | 6. Finds the components of displacement of a projectile in a given time. | Derive that the components of displacement at time $t$ as $x = (u\cos\alpha)\cdot t$ and $y = (u\sin\alpha)\cdot t - \frac{1}{2}gt^2$ .<br>State that these are parametric equations of the path of the projectile where "$t$" is the parameter. | |
| | 7. Calculates the maximum height of a projectile. | Show that if H is the maximum height, then $H = \dfrac{u^2\sin^2\alpha}{2g}$ | |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
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| | 8. The time taken to reach the maximum height of a projectile. | Show that if T is the time taken to reach the maximum height, then $T = \frac{u \sin \alpha}{g}$. | |
| | 9. Calculates the horizontal range of a projectile. | If $T'$ is the time taken to return to the level of the point of projection. Then show that $T' = \frac{2u \sin \alpha}{g} = 2T$. | |
| | 10. Calculates the horizontal range of a projectile. | If R is the horizontal range through the point of prejection derive the expression $R = \frac{2u^2}{g} \cos \alpha \sin \alpha$ | |
| | 11. State that in general there are two angles of projection for the same horizontal range for a given velocity of projection. | Show that there are two angles of projection giving the same value for R which is obtained when $\alpha = \theta$ and $\alpha = \frac{\pi}{2} - \theta$ are substituted in the the expression for the same horizontal range $R = \frac{2u^2}{g} \cos \alpha \sin \alpha$. | |
| | 12. Finds the maximum horizontal range for a given speed. | As $R = \frac{u^2 \sin 2\alpha}{g} \leq \frac{u^2}{g}$ deduce that $R_{max} = \frac{u^2}{g}$, for given $u$. | |
| | 13. For a given speed of projection finds the angle of projection giving the maximum horizontal range. | Derive that the angle of projection giving the maximum horizontal range is $\frac{\pi}{4}$. | |
| | 14. Derives Cartesian equations of the path of a projectile. | Derive the equation $y = x \tan \alpha - \frac{gx^2 \sec^2 \alpha}{2u^2}$ by eliminating *t* from the equation obtained earlier for *x* and *y* when $\alpha \neq \frac{\pi}{2}$. Compare this with the familiar quadratic function $y = ax - bx^2$. Remind that $\alpha = \frac{\pi}{2}$ gives vertical motion under gravity. | |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
|---|---|---|---|
| 3.9 | 1. States Newton's first law of motion. | A body continues to be in a state of rest, or of uniform motion along a straight line, unless it is a acted upon by an external force. | 15 |
| | 2. Defines "force". | Define 'force' as the external action that changes the state of motion of a body. | |
| | 3. Defines "mass". | Define mass as the amount of response of the body to a force acting on it. | |
| | 4. Defines linear momentum of a particle. | Define linear momentum of a particle of mass *m* moving with a velocity *u* as *mu*. | |
| | 5. States that linear momentum is a vector quantity. | Show that in the product *mu*, the velocity is a vector and hence the linear momentum is also a vector. | |
| | 6. States the dimensions and unit of linear momentum. | The dimensions of linear momentum are $\left[MLT^{-1}\right]$. Unit of linear momentum is $kgms^{-1}$. | |
| | 7. Describes an inertial frame of reference. | Describe an inertial frame of reference as a frame, which is at rest or moving with a uniform velocity relative to a frame of reference fixed on the earth. [Earth is considered as an inertial frame of reference in studying motions taking place on Earth's - surface.] | |
| | 8. States Newton's second law of motion. | Tha rate of change of linear momentum of a body is directly proportional to the external force. Derive $F = k\ ma$ from the second law. | |
| | 9. Defines Newton as the absolute unit of force. | The Newton is defined as the force required to produce an acceleration of 1 $ms^{-2}$, on a mass of 1 $kg$. | |
| | 10. Derives the equation $\underline{F} = m\underline{a}$ from second law of motion. | From Newton's second law of $F = k\ ma$ using to the definition of the Newton show that $k = 1$. Therefore $F = ma$. Hence *m* is measured in kilograms *a* in meters per second per second $\left(ms^{-2}\right)$ then F is measured in Newtons. | |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
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| | 11. Explains the vector nature of the equation $\underline{F} = m\underline{a}$ from second law of motion. | According to $\underline{F} = m\underline{a}$ the acceleration takes place in the direction of the force F. Show that the equation could be used by resolving the force and the acceleration in any direction. | |
| | 12. States the gravitational units of force. | Introduce *kg* weight as the gravitational force with which a mass of 1*kg* is pulled towards the earth, gravitational attraction. | |
| | 13. Explains the difference between mass and weight of a body. | Introduce that the mass of a body is the amount of matter contained in it and that the weight of a body is the graviational force with which that body is attracted towards the earth. State that when the mass is measured in kg, and the acceleration due to gravity is measured in $ms^{-2}$ the units of weight is obtained in N. | |
| | 14. Describes "action" and "reaction". | Draw the attention of students to different types of forces - using examples show how forces always occur in pairs. For example, action and reaction between two bodies in contact. | |
| | 15. States Newton's third law of motion. | "For every action between bodies, there is a reaction of equal magnitude and opposite direction along the same line of action" | |
| | 16. Solves problems using $\underline{F} = m\underline{a}$ . | Students are expected to solve problems related to following cases.<br>i. Find acceleration of a body under the action of given external forces. Find external force when the acceleration is given . Find reaction between body and a lift when the lift is moving with given acceleration. The acceleration produced by external forces acting on two bodies connected by a string and the tension in the string.<br>ii. When forces act on a system of bodies., find the acceleration of each body of the system and reactions between bodies in the system.<br>iii. Calculate frictional forces acting, by considering the motion of a body on a rough plane.<br>iv. Problems relating to the motion of systems comprising of connected particles or rigid bodies.<br>v. Motion of particle on a smooth wedge, which may (including smooth pulleys) moving on a smooth plane. | |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
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| 2.8 | 1. Describes "friction" and "Frictional force". | Introduce frictional force as a force acting along the tangential plane that prevents or tends to prevent relative motion between two bodies in contact. The property that produces this force is called friction. State that as the applied force is gradually increased the frictional force too increases until the equilibrium ceases to exist. | 10 |
| | 2. Distinguishes between smooth and rough surfaces. | From the surfaces under impact, introduce that the surfaces without a frictional force as smooth surfaces and surfaces with a frictional force as rough surfaces. | |
| | 3. States the advantages and disadvantages of friction. | Discuss the advatages and disadvantages of friction using suitable examples. | |
| | 4. Describes "Limiting frictional force". | Introduce "Limiting frictional force" as the maximum frictional force between two surfaces at the instant when relative motion between the two surfaces in contact is about to commence. | |
| | 5. State laws of friction | (1) When two bodies on contact undergo motion relative to each other the frictional force exerted by one body on the other is opposite to the direction in which the body moves or tends to move relative to the other.<br>(2) When in equilibrium the frictional force is just sufficient to prevent relative motion.<br>(3) The ratio of the limiting frictional force to the normal reaction at the point of contact is a constant, called the coefficient of limiting friction and depends only on the material the surface of the bodies are made of.<br>(4) As long as the normal reaction remains unchanged, the limiting frictional force does not depend on the area of contact or the shape of the two surfaces.<br>(5) When the motion sets in the limiting frictional force decreases slightly.<br>(6) When there is relative motion between the surfaces, the ratio of the frictional force and the normal reaction is slightly less than the coefficient of limiting friction. | |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
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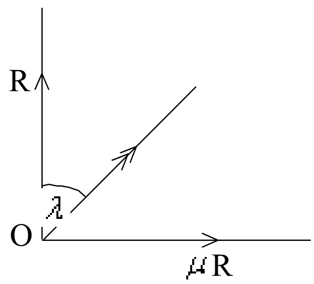
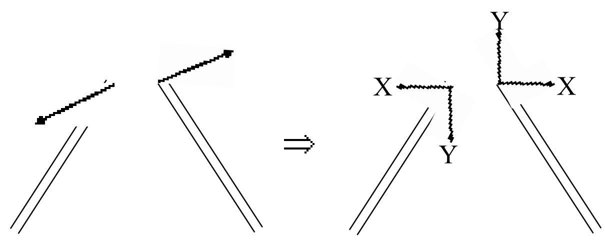
| | 6. Defines coefficient of limiting friction. | Define the coefficient of limiting friction between any two given surface as the ratio of the limiting frictional force to the normal reaction. If $F_l$ is the limiting frictional force and R is normal reaction then $\mu_l = \frac{F_l}{R}$ is called coefficient of limiting friction. This is also called the coefficient of static friction. | |
| | 7. Introduces angle of friction. | Introduce the angle of friction as the angle made by the total reaction with the normal reaction when the equilibrium is limiting.<br>If the angle of friction is $\lambda$; deduce the result $\mu_l = \tan\lambda$<br> | |
| | 8. Expresses the conditions for equilibrium. | Show that the frictional force F between two surfaces of contact at any time is given by $F \le \mu_l$. (equality occuring at limiting equilibrium) and hence $F \le \mu_l R$ . | |
| | 9. Solves problems related to equlibrium of a particle or a rigid body, under the action of forces including frictional forces. | Direct students to solve friction related problems. | |

<table>
<tr><th>Com. Level</th><th>Learning Outcomes</th><th>Guidelines for Subject Matter</th><th>No. of Periods</th></tr>
<tr><td>2.9</td><td>1. Describes differenct types of joints.</td><td>Joints<br>- Movable Joints<br>  - Bowl Joints<br>    - Smooth<br>    - Rough<br>- Rigid Joints<br>  - Hinges (Nut and Bolt Joints)<br>    - Smooth<br>    - Rough<br><br>Discuss each type of joint by considering examples in every day life.</td><td>10</td></tr>
<tr><td></td><td>2. Describes the difference between movable joint and rigid joint.</td><td>Introduce that<br>(i) A joint where the position of rods cannot be changed relative to one another as a rigid joint.<br>(ii) A joint where the position of rods can be changed relative to one another as a movable joint.</td><td></td></tr>
<tr><td></td><td>3. Describes smooth hinge (Nut and Bolt joint)</td><td>State that the hight or heavy rods, considered are joined by smooth hinges (Nut and Bolt joints). Show that since the joint is smooth the force acting at the joint act in the plane of the two rods and that the reactions between the rods under the action of external forces are equal in magnitude and opposite in direction.<br></td><td></td></tr>
<tr><td></td><td>4. Solves problems involving smooth joints.</td><td>Direct students to solve relevant problems.</td><td></td></tr>
</table>

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
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| | 5. Solves problems dealing with rigid joints. | Emphasize that the equilibrium cannot be considered as that of two bodies by disjoining the joint. Show that solving problems involving two rods jointed rigidly has to be considered as single rigid body.<br>(Rough hinges are not within the syllabus) | |
| 2.10 | 1. Introduces a frame work with light rods. | Introduce a fram work as a structure consisting of three or more straight light rods hinged at their ends such that they are in the same plane.<br>Describe using suitable examples. | |
| | 2. States the assumptions made in considering frame works. | (i) All the rods are smoothly joined at their ends so that no couple or torque is produced.<br>(ii) Except for the external forces, all the reactions at the joints act along the rods. These can be either tensions or thrusts.<br>(iii) All the rods lie in the same vertical plane. Therefore all the forces in the system including external forces are coplanar. | |
| | 3. States the conditions necessary for the equilibrium of each joint. | (i) As the entire frame work is in equilibrium, all the external forces should be in equilibrium.<br>(ii) As each joint is in equilibrium under the action of forces acting on it (external forces and the stress meeting in the joint) the principle of polygon of forces can be applied to each joint. | |
| | 4. When the frame work is symmetrical marks the external forces acting on it. | Show using suitable examples how the symmetrical property of the system is used in marking the forces and how to use conditions of equilibrium in finding them. | |
| | 5. States Bow's notation. | Tell the students that, as this method was found by a Mathematician by the name of "Bow" the method is called "Bow's notation". | |

| Com. Level | Learning Outcomes | Guidelines for Subject Matter | No. of Periods |
|---|---|---|---|
| | | Method:-<br>(i) Mark the external forces acting on a frame work, outside this frame work itself.<br>(ii) Number all areas bounded by every pair of external forces and rods. | |
| | 6. Draws the stress diagram for the frame work. | (i) Draw a triangle of forces, or a polygon of forces representing forces at a joint, with one or two unknown forces. (these force diagram should be a closed polygon).<br>(ii) Draw triangles or polygons of forces for adjoining joints taken in order. These figures too should be closed. | |
| | 7. Calculates the stresses by the method of Bow's notation. | (i) Find unknown stresses by considering the triangles or polygons in the stress diagram, and using trigonometric or algebraic formulae.<br>(ii) Show that (if necessary) unknown external forces can also be determined, using the stress diagram.<br>• Draw the attention of students to avoid using arrow heads on the stress diagram, but use these on the frame work diagram.<br>• Explain how the stress could be classified as tensions and thrust according to the arrow heads inserted on the frame work diagram. | |

# Combined Maths

## Grade 12

## Teacher's Instruction Manual

School Based Assessment

# Introduction- School Based Assessment

Learning -Teaching and Evaluation are three major components of the process of Education. It is a fact that teachers should know that evaluation is used to assess the progress of learning-teaching process. Moreover, teachers should know that these components influence mutually and develop each other. According to formative assessment (continuous assessment) fundamentals; it should be done while teaching or it is an ongoing process. Formative assessment can be done at the beginning, in the middle, at the end and at any instance of the learning teaching process.

Teachers who expect to assess the progress of learning of the students should use an organized plan. School based assessment (SBA) process is not a mere examination method or a testing method. This programme is known as the method of intervening to develop learning in students and teaching of teachers. Furthermore, this process can be used to maximize the student's capacities by identifying their strengths and weaknesses closely.

When implementing SBA programmes, students are directed to exploratory process through Learning Teaching activities and it is expected that teachers should be with the students facilitating, directing and observing the task they are engaged in.

At this juncture students should be assessed continuously and the teacher should confirm whether the skills of the students get developed up to expected level by assessing continuously. Learning teaching process should not only provide proper experiences to the students but also check whether the students have acquired them properly. For this, to happen proper guiding should be given.

Teachers who are engaged in evaluation (assessment) would be able to supply guidance in two ways. They are commonly known as feed-back and feed- forward. Teacher's role should be providing Feedback to avoid learning difficulties when the students' weaknesses and inabilities are revealed and provide feed-forward when the abilities and the strengths are identified, to develop such strong skills of the students.

Student should be able to identify what objectives have achieved to which level, leads to Success of the Learning Teaching process. Teachers are expected to judge the competency levels students have reached through evaluation and they should communicate information about student progress to parents and other relevant sectors. The best method that can be used to assess is the SBA that provides the opportunity to assess student continuously.

Teachers who have got the above objective in mind will use effective learning, Teaching, evaluation methods to make the Teaching process and learning process effective. Following are the types of evaluation tools student and, teachers can use. These types were introduced to teachers by the Department of Examination and National Institute of Education with the new reforms. Therefore, we expect that the teachers in the system know about them well

Types of assessment tools:

1. Assignments
2. Projects
3. Survey
4. Exploration
5. Observation
6. Exhibitions
7. Field trips
8. Short written
9. Structured essays
10. Open book test

11. Creative activities
12. Listening Tests
13. Practical work
14. Speech
15. Self creation
16 Group work
17. Concept maps
18. Double entry journal
19. Wall papers
20. Quizzes
21. Question and answer book
22. Debates
23. Panel discussions
24. Seminars
25. Impromptus speeches
26. Role-plays

Teachers are not expected to use above mentioned activities for all the units and for all the subjects. Teachers should be able to pick and choose the suitable type for the relevant units and for the relevant subjects to assess the progress of the students appropriately. The types of assessment tools are mentioned in Teacher's Instructional Manuals.

If the teachers try to avoid administering the relevant assessment tools in their classes there will be lapses in exhibiting the growth of academic abilities, affective factors and psycho- motor skills in the students

# Grade 12 First Term - Evaluation Plan - 1
## (Combined Maths - I)

**01. Competency** : 15. Derives relations involving measurement of angles to solve problems.
**Competency Level** : 15. Derives relations involving measurement of angles to solve problems.

**02. Nature of Evaluation Plan** : Group activity of finding the length of an arc of a circle and the area of a sector.

**03. Time** : 100 minutes

**04. Instructions for the implementation of the evaluation plan :**

Prepare
(1) Copies of work sheets in annexure 1
(2) Protractor
(3) Thread
(4) Pins
(5) Ruler

**Step 1** :
(i) Divide the students into two groups A and B.
(ii) Provide the group with copies of worksheet (1).
(iii) Direct the students to present the explanation to the class.
(iv) Prepare the students to present the exaplanation to the class.

**Step 2** :
(i) Divide the students into two groups A and B.
(ii) Provide the group with copies of worksheet (2).
(iii) Direct the students to engage in the activities as instructed.
(iv) Prepare the students to present the exaplanations to the class.

**05. Criteria for Evaluation :**

1. Expressing the arc length and the area of the sector subtended by it in terms of the radius and the radians in the angle it subtends at the centre.
2. Realises that the arc length and the area of the sector can be determined when the radius $r$ of the circle and the angle subtends at the centre are known.
3. Calculating arc length and the area of the sector when the radius of the circle and the angle subtended at the centre are given.
4. Inclination for the exploration of suitable mathematical models required for calculations arising in day to day life.
5. Selecting suitable solutions in solving problems.

## Work Sheet (1)

- Your group is required to engage in the activity according to the instructions given. Be prepared to present the explanations to the class at the end of the activity.

**Group A**

(i) Draw the line OA 5cm long.

(ii) With centre as O, draw 4 circles with radius 2cm, 3cm, 4cm and 5cm.

(iii) Mark the points of intersection of the circles with the line OA as P, Q and R.

(iv) Taking a piece of thread marked 2cm on it mark points 2cm from P on the circumlerence of the circle with radius 2cm: mark those points as $P_1$.

(v) Mark the points of intersection of other circles when $OP_1$ is produced as $Q_1$ and $R_1$.

(vi) Measure the lengths of the arcs $QQ_1$ and $RR_1$.

(vii) Measure the angle $QOR_1$.

(viii) What can you say about the angle subtended by an arc of any circle equal to its radius?

**Group B**

(i) Draw a line segment OA, 4cm long and draw a circle with radius OA.

(ii) Taking a piece of thread marked 4cm on it mark the point P, 4cm from A on the circumference. Join that point to O. Measure the angle obtained.

(iii) Mark two angles equal to $P\hat{O}A$ away from OA and OP.

(iv) If Q and R are the points where the sides of those angles meet the circle measure the arcs AQ and PR.

(v) What can you say about the arc lengths when the angles subtended at the centre of a circle are equal?

## Work Sheet (2)

- Your group is required to engage in the activity according to the given instructions. Be prepared to present explanations to the class at the end of the activity.
- Group A should complete the tables (1) and (2) and Group B should complete tables (1) and (3). Answer the given questions using the tables.

**Table (1)**

| Radius of Circle | Angle subtended at the centre | Length of the arc subtending that angle |
|---|---|---|
| $r$ | 1 Radian | ............................ |
| $r$ | 3 Radians | ............................ |
| ....... | 5 Radians | ............................ |
| 2 | ....... Radians | ............................ |
| $r$ | $\theta$ Radians | ............................ |

**Table (1)**

| Radius of Circle | Angle subtended at the centre | Length of the arc subtending that angle |
|---|---|---|
| ....... | 1 Radian | $r$ |
| ....... | $\theta$ Radians | $r\theta$ |
| $r$ | ....... Radians | $2\pi r$ |
| $r$ | ....... Degrees | $2\pi r$ |
| 4 | 60 Degrees | ............................ |

**Table (3)**

| Radius of Circle | Angle subtended at the centre | Area of a sector of a circle subtending that angle |
|---|---|---|
| ....... | $360^0$ | $\pi r^2$ |
| $r$ | $2\pi$ | $\frac{1}{2}(.......)r^2$ |
| ....... | $\pi$ | ............................ |
| ....... | $\theta$ | ............................ |
| ....... | $\frac{\pi}{3}$ | ............................ |

- Denote a relation between degrees and radians.
- Write P in radians.
- Write the value of a radian in degrees.

# Grade 12 First Term - Evaluation Plan - 2
# (Combined Maths - I)

**01. Competency :** 18. Applies sine formula and cosine formula to solve trigonometric problems.

**Competency Level :** 18. Applies sine formula and cosine formula to solve trigonometric problems.

**02. Nature of Evaluation Plan :**

It is an assignement which can be implemented for the main groups under the theme sine rule and cosine rule.
It is expected to obtain a knowledge about the relationship between the length of the sides and trogonometric ratios in triangular figures of various shapes.
The final outcome will be a table consisting practical values denoting that relation.

**03. Time :** 90 minutes

**04. Instructions for the implementation of the evaluation plan :**

(a) Pre-requisites and instructions.

(i) Provide the students with the previous knowledge required to implement the assignment. Discuss with students about practical situations involving the finding of lengths of sides of a triangle.
(ii) Divide the students into groups.
(iii) Prepare work sheets required for the groups. When there are several groups provide them with parallel work sheets.
(iv) Make the students aware of the materials required for the task assigned to each group and the evaluation criteria.
(v) Prepare materials for the two groups as follows.
A white A4 paper, pencil, eraser, two pieces of white thread, a meter scale, a protractor, a wooden board to fix the white paper, drawing pins, two nails about 3cm long.

(b) Procedure

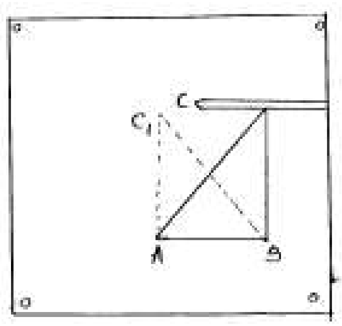
(i) Providing each group with a work sheet and engage them in the activity.
(ii) Watch the groups and give instructions and guidance to make the assignment a success.

## Work Sheet for Group 1

You are required to measure sides of various triangles with sides of different length and to test that they agree with the sine rule and the cosine rule.

(i) Read this work sheet and discuss among you how to accomplish the task assigned to you.

(ii) Fix the A4 paper to the wooden board with pins

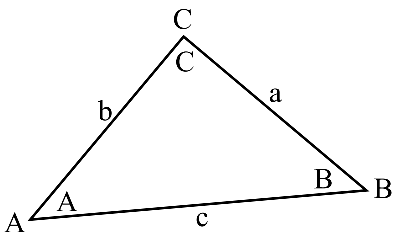



(iii) On the lower end of the paper draw a lline AB of sufficient length parallel to the lower edge. Measure the length of AB. Fix firmly the two nails on the vertices A and B. Tie two unequal pieces of thread to A and B. Join the free ends of the threads to a pencil. If C is the pencil point take the length of BC less than AB. AC and BC are the two pieces of thread and C is the pencil point.

(iv) Initially place the pencil point on the paper with the thread BC perpendicular to AB and with the threads taut and mark the point $C_1$. Similarly changing the length of AB mark at least 5 positions for C with the threads taut, on the plane of the paper rotating the point C anticlockwise. Takes cases for $C\hat{A}B < \frac{\pi}{2}$ only. Measure AC and BC. (obtain the length of BC varying the length AC).

(v) Trace the edges AC and CB for variuos positions of C and measure $C\hat{A}B$ and $C\hat{B}A$ for each position.

(vi)




Complete the following table taking $C\hat{A}B = \hat{A}$, $C\hat{B}A = B$, $AC = b$ and $CB = a$, $AB = c$.

(vii) $AB = c$, $C = 180 - (A + B)$

**Table 1**

| AC<br>b | BC<br>a | $\frac{a}{\sin A}$ | $\frac{b}{\sin B}$ | $\frac{c}{\sin C}$ | $\cos A$ | $\frac{b^2+c^2-a^2}{2bc}$ |
|---|---|---|---|---|---|---|
| | | | | | | |

(viii) Give your conclusions about the ratios

$\frac{a}{\sin A}$, $\frac{b}{\sin B}$, $\frac{c}{\sin C}$.

(ix) What is your conclusion about $\cos A$ and $\frac{b^2+c^2-a^2}{2bc}$.

**Work Sheet for Group 2**

Work sheet will be the same as for Group 1, but take the magnitudes of sides and angles for $C\hat{A}B > \frac{\pi}{2}$.

## 05. Criteria for Evaluation :

1. It is expected to evaluate the preparation, the finish of the procedure and the traits of the students with regard to the assignment.
2. Group mark or personal marks may be given for the criteria in the evaluation model.
3. Assign marks for each criterion as follows,

   Excellent - 4 Very good - 3

   Good - 2 Average - 1
4. Accordingly the maximum number of marks entitled to a student is $14 \times 4 = 56$.

## Evaluation Model

(01) Preparation
- (i) Taking part in the discussion brilliantly.
- (ii) Exchange of ideas with the teacher.
- (iii) Exchange of ideas with other students.
- (iv) Contribution towards planning the activities.

(02) Engaging in the process
- (i) Selecting lengths AB, AC, BC suitably.
- (ii) Selecting lengths AB and BC suitably.
- (iii) Measuring triangles

(03) Finish
- (i) Preparation of the table
- (ii) Conclusion about ratios .
- (iii) Conclusion about and .

(04) Traits exhibited in the student from the begining to the end of the assignment.
- (i) Obeying teacher instructions.
- (ii) Interest in taking measurements.
- (iii) Co-operation with the group.
- (iv) Ability to tolerate opinions of others.

## Grade 12 First Term - Evaluation Plan - 3
## (Combined Maths - II)

**01. Competency** : 1. Manipulates Vector Algebra.
**Competency Level** : 1.3 Applies position vector as a technique of solving problems.

**02. Nature of Evaluation Plan** : Solving problems using position vectors is a group activity.

**03. Time** : 90 minutes

**04. Instructions for the implementation of the evaluation plan :**

Get supplie with following materials.

(i) Three copies of the work sheet in annexure 1.
(ii) Demi papers
(iii) Marker pens

**Step 1 :** (i) Divide the students into three groups A, B and C.
(ii) Prepare the students to present the explanations to the class.

**05. Criteria for Evaluation :**

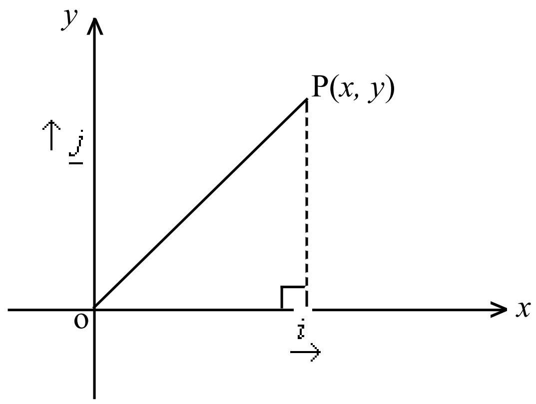
1. Expressing what the position vector of a point is.
2. Realising that by using position vectors, the position of any point in relation to a given point can be determined distinctly.
3. Applying the information studied to solve problems.
4. Communication of information using various techniques.
5. Using previous experience to obtain new experience.

## Work Sheet

- Get engaged in the task assigned to your group.
- Be prepared to present group explanations to the whole class.

### For all groups

The unit vector along the $x$-axis is $\underline{i}$ and the unit vector along the $y$-axis is $\underline{j}$. The position of a point $P(x, y)$ in relation to a system of axes $ox$, $oy$ is denoted in the following diagram.




Obtain that $\overrightarrow{OP} = x\underline{i} + y\underline{j}$

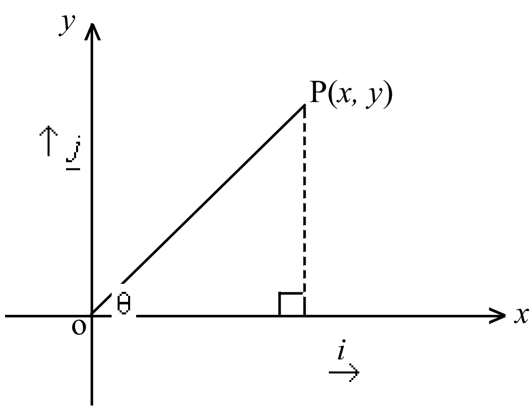
### For Group A

(a) Consider that the position vector of A is $\overrightarrow{OA} = \underline{a}$ and the position vector B is $\overrightarrow{OB} = \underline{b}$ in relation to a point O. Denote these data in a diagram. Obtain an expression in $\underline{a}$ and $\underline{b}$ for $\overrightarrow{AB}$.

(b) Deduce an expression for $\overrightarrow{BA}$.

### For Group B

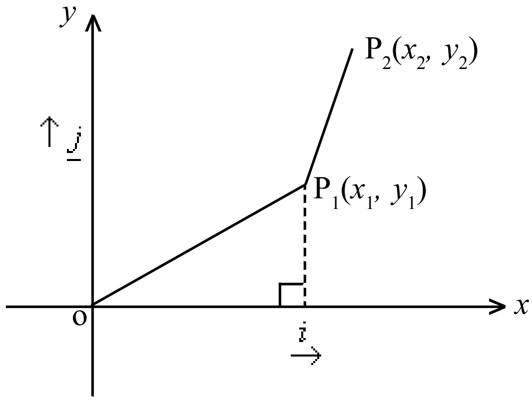
(a) Consider the point P($x$, $y$).



(i) Express $\overrightarrow{OD}$, $\overrightarrow{DP}$ and $\overrightarrow{OP}$ in terms of $\underline{i}$ and $\underline{j}$.

(ii) Express the unit vector along the vector $\overrightarrow{OP}(\underline{u})$ in terms of $\underline{i}$, $\underline{j}$ and $\theta$ only.

**For Group C**




(i) Express the position vectors of the points $P_1$ and $P_2$ denoted in the diagram in terms of the unit vectors $\underline{i}$ and $\underline{j}$ and the co-ordinates of those points.

(ii) Hence find $\overrightarrow{P_1P_2}$.

(iii) Express the unit vector along $\overrightarrow{P_1P_2}$.

## Grade 12 Second Term - Evaluation Plan - 1
## (Combined Maths - I)

**01. Competency** : 3. Analyses Quadratic Functions.
**Competency Level** : 3.1 Investigates the properties of a quadratic function.

**02. Nature of Evaluation Plan** : It is a group activity of finding the properties of a quadratic function.

**03. Time** : 120 minutes

**04. Instructions for the implementation of the evaluation plan :**

Get supplied with
- (i) Six copies of the instructions sheet in annexure 1
- (ii) A bristol board prepared according to the note in annexure 2.
- (iii) Demi papers
- (iv) Marker pens

**Step 1** :
- (i) Divide the class into six groups.
- (ii) Supply each group with a copy of the worksheet.
- (iii) Get the groups engaged as instructed in the work sheet.
- (iv) Prepare the groups to present their exaplanation to the class.

**05. Criteria for Evaluation :**

1. Describing the properties of the quadratic function.
2. Realises that the shape of a quadratic function can be determined by the properties of $a$ and $\Delta$.
3. Determining the behaviour of a quadratic function by considering the coefficient of the square term and its discriminant.
4. Activity with co-operation with in the group.
5. Derivation of more efficient methods for prediction.

## Work Sheet

- You are required to choose the quadratic function relevant to your group from the following functions.

| | | |
|---|---|---|
| Group A | - | $f(x) = 2x^2 - 4x + 5$ |
| Group B | - | $f(x) = -3x^2 + 2x - 1$ |
| Group C | - | $f(x) = x^2 - 3x + 2$ |
| Group D | - | $f(x) = -2x^2 + 3x + 5$ |
| Group E | - | $f(x) = x^2 - 2x + 1$ |
| Group F | - | $f(x) = -x^2 + 2x - 1$ |

- Do the following activity according to the given instructions.

(i) For the given function $y = f(x)$ complete the following table.

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | | |

(ii) Draw the graph of $y = f(x)$ for the interval $-3 \leq x \leq 4$.

(iii) Using the graph drawn answer the following questions.

(a) What is the greatest or the least value of the function?

(b) Write the equation of the symmetrical axis of the graph.

(c) Write the co-ordinates of the vertex of the graph.

(d) Are there points where the $y$ co-ordinate is zero? If so, write their $x$ co-ordinates.

(e) By completing the square write the given function in the form $f(x) = A(x + B)^2 + C$; $A, B, C \in \mathbb{R}$.

(f) Using the result obtained in (e) answer the questions (a), (b), (c) and (d) above.

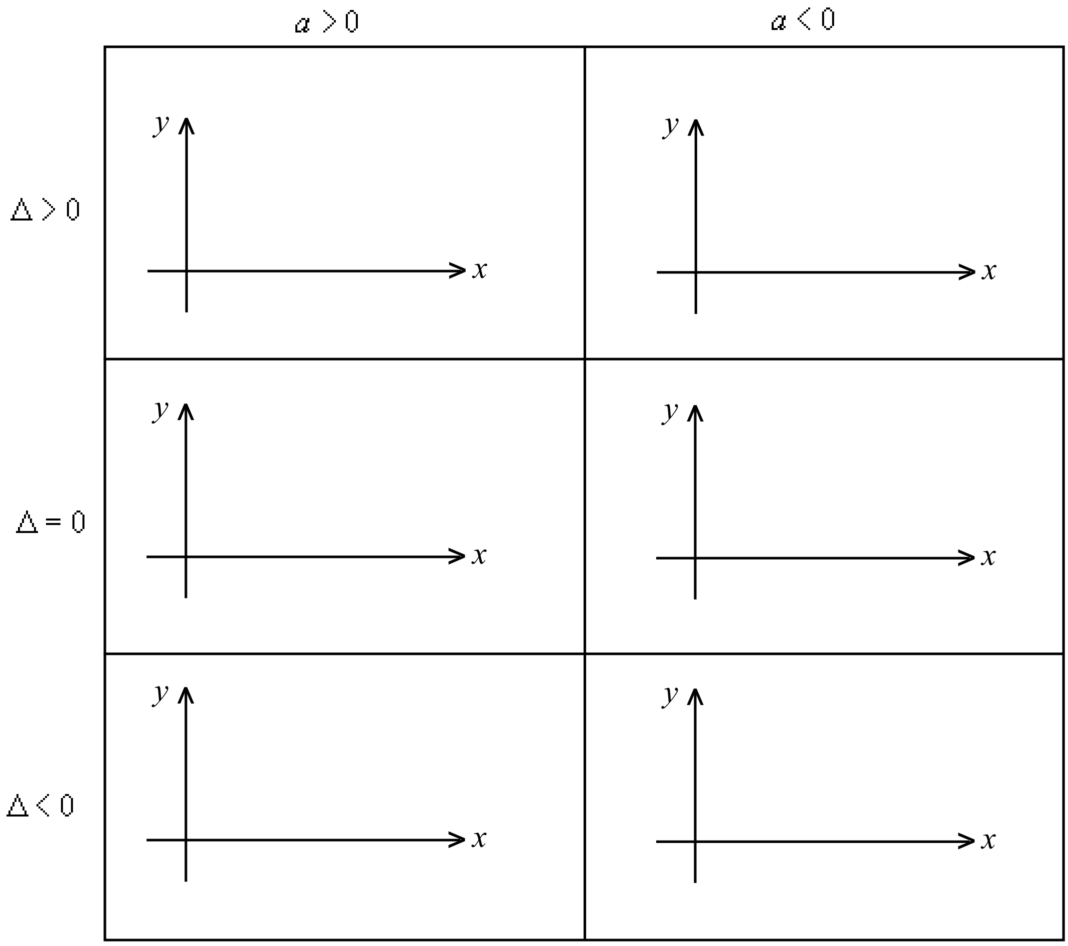
(g) When the given function is compared with the standard form $f(x) = ax^2 + bx + c$. What is the sign of $a$?

(h) $b^2 - 4ac$ in the general quadratic function is called the discriminant. It is generally denoted by $\Delta$ (delta). Write the value of $\Delta$ for the given function.

(i) What is the sign of $\Delta$?

- Mark the graph drawn at the appropriate place of the poster hung in the class.

Model for making the explanations (Graphs)



# Grade 12 Second Term - Evaluation Plan - 2
# (Combined Maths - II)

**01. Competency** : 3. Perceives natural motions on a plane using the Newtonians model for motion

**Competency Level** : 3.1 Use graphs to solve problems involving the motion on a straight line.

**02. Nature of Evaluation Plan** :

(i) It is a group activity which can be implemented under the theme of motion along a straight line.

(ii) Can realise the difference between displacement and distance, average velocity and average speed.

(iii) Find outcome will be a table of numerical values obtained for average speed and average velocity and a wall newspaper with displacement - time and distance - time graphs.

**03. Time** : 90 minutes

**04. Instructions for the implementation of the evaluation plan :**

(a) Prerequisites and Instructions.

(i) On a day before implementing the assignment discuss with the students about practical situations and their aims.

(ii) Prepare work sheets required for the groups.

(iii) Make groups with 5 to 6 students. When there are several groups give parallel work sheets.

(iv) Create an awareness among students about the materials and information that have to be supplied according to the task assigned to each group and the evaluation criteria.

(v) Prepare materials for each group as follows.

Materals and information for group 1

Piece of chalk, meter scale, four stop watches, rubber ball or a piece of wood.

Materals and information for group 1

Piece of chalk, meter scale, five stop watches, rubber ball, a plank that can be used as a barrier.
(even a wall is suitable for this purpose)

(b) Procedure

(i) Engage each group in the activity by providing them with relevant work sheets.

(ii) Watch the groups in activity and give them necessary instructions for its success.

(iii) Give opportunity to present their creations to the class.

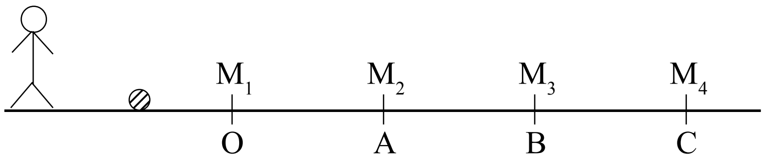
**Work Sheet for Group 1**

- Your group is required to measure the displacement and distance of a particle moving on a straight line and to calculate the average speeds and velocities in the given time intervals. This can be accomplished by projecting a ball or an object through four points in the cemented floor of your school compound and measuring the time taken to pass each point.

(i) Discuss the task assigned with the members in the groups.

(ii) Provide a ball, meter scale, four stop watches and 2 graph papers for each member.

(iii) Mark 4 points on the cemented floor in the school compound from 3m-4m apart by the piece of chalk. Name these points as O, A, B and C. The distances between them need not be equal.




(iv) Place four students $M_1$, $M_2$, $M_3$ and $M_4$ with stop watches in front of the points O, A, B and C.

(v) From a sufficient distance from O project the rubber ball along the floor with a suitable velocity to pass through the 4 points.

(vi) At the moment the ball is projected all students should activate their stop watches.

(vii) As the ball reaches each point the student beside that point should stop the watch and obtain the relevant reading.

(viii) Measure the distance from O to A, B and C.

(ix) Tabulate your readings as follows.

| Displacement | time taken for the journey |
|---|---|
| OA = .......... | $t_{OA}$ = time taken to travel from O to A |
| OB = .......... | $t_{OB}$ = ............................................... |
| OC = .......... | $t_{OC}$ = ............................................... |

(x) Draw the *s-t* graph.

(xi) Use, Average velocity $= \dfrac{\text{Difference in displacement}}{\text{Difference in time}}$

and average speed $= \dfrac{\text{Difference in distance}}{\text{Difference in time}}$

to complete the following table.

| Intervals | Average velocity | Average speed |
|---|---|---|
| OA | | |
| OB | | |
| OC | | |
| AB | | |
| BC | | |

(xii) Discuss your results and conclusions with other groups.

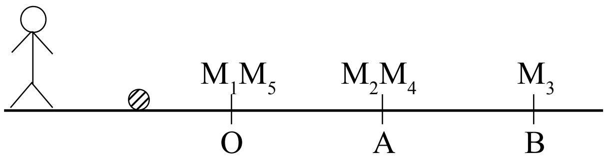
**Work Sheet for Group 2**

- Your group is required to measure the displacements and distances between some points taken on a straight line and to calculate the average velocity and average speed of a body passing through the points. You are required to measure the displacements and distances between several selected points and measure the time taken to make those displacements by a ball passing through them.

(i) Discuss the task assigned to you with other members in the groups.

(ii) Obtain a ball, meter scale to measure displacement and distance, 5 stop watches to measure time, a piece of chalk, a plank or a similar object which can be used as a barrier and two graph papers for each member in the group.

(iii) Mark three points O, A and B about 3m - 4m apart on the cement floor in the school compound using the piece of chalk. Their distances may not be equal. Place the barrier at B perpendicular to OAB. (A wall may be used as the barrier)




(iv) Place five students with stop watches as follows.

(a) Two students $M_1$ and $M_5$ with stop watches.

(b) Two students $M_2$ and $M_4$ at A.

(c) One student $M_3$ at B.

(v) Projet the ball along the ground from a sufficient distance from O with a suitable velocity to pass through O, A and B and after hitting the barrier at B to reach O or pass through O.

(vi) Immediately after projecting the ball all students should activate their stopwatches.

(vii) When the ball travels through O, A and B, $M_1$, $M_2$ and $M_3$ should stop their stopwatches, when B reaches A and O after hitting B, $M_4$ and $M_5$ should stop their stopwatches.

(viii) Obtain the readings of the stopwatches.

(ix) Measure distances from O to A and B.

(x) Tabulate the values obtained as follows.

(xi) From the values obtained above draw a displacement-time curve and a distance - time curve.

| | Displacement | Distance | Time |
|---|---|---|---|
| Journey OA<br>,, OB<br>,, OBA<br>,, OBAO | | | |

**05. Criteria for Evaluation :**

1. It is expected to evaluate the readiness, finish and the traits of students towards the activity. Use the model given for it.
2. Assign marks for the criteria in the evaluation model group - wise or individual student - wise as circumstances demand. Group marks will be the individual mark of the student for each criterion.
3. Assign marks for each criterion as follows.
   Excellent - 4 Very good - 3
   Good - 2 Average - 1
4. Accordingly the maximum mark a student entitled to is $12 \times 4 = 48$.

## Evaluation Model

| Criteria | Name of Student | |
|---|---|---|
| | Group 1 | Group 2 |
| (01) Preparedness<br>(i) Energetic participation in discussion.<br>(ii) Exchange of ideas with the teacher.<br>(iii) Exchange of ideas with other students.<br>(iv) Contribution towards planning the activities.<br><br>(02) Engaging in the activity<br>(i) Selecting O, A, B, C suitably.<br>(ii) Throwing the ball appropriately.<br>(iii) Measuring time.<br>(iv) Measuring displacement.<br><br>(03) Finish<br>(i) Preparing table 1.<br><br>(04) Traits exhibited by the student from the begining to the end of the activity.<br>(i) Following teacher instructions.<br>(ii) Interest shown in taking measurements.<br>(iii) Co-operation with the group. | | |

# Grade 12 Second Term - Evaluation Plan - 3
# (Combined Maths - II)

**01. Competency** : 3. Perceives natural motions taking place on a plane using the Newtonian model for motion.

**Competency Level** : 3.1 Uses kinetic equations to solve problems on motions along a straight line.

**02. Nature of Evaluation Plan** : It is a group activity of constructing equations for motion.

**03. Time** : 80 minutes

**04. Instructions for the implementation of the evaluation plan :**

1. Four copies of the work sheet in annexure 1.
2. Demi papers.
3. Maker pens, should be supplied.

**Step 1 :**
- (i) Divide the class into 4 small groups.
- (ii) Provide each group with a copy of the work sheet.
- (iii) Distribute demi papers, marker pens and copies of the work sheet among groups.
- (iv) Get the groups engaged in the activities.
- (v) Prepare the students to present explanations to the class.

**05. Criteria for Evaluation :**

1. Construction of kinetic equations on motion.
2. Realising that application of kinetic equations in convenient in solving some problems.
3. Solving problems using kinetic equations.
4. Exploring alternate methods for solving problems.
5. Facilitating the task by constructing models.

**Work Sheet**

- Your are required to engage in the activity according to given instructions.
- Select the process relevant to your group. Hence construct the following equations.

(i) $v = u + at$

(ii) $s = ut + \frac{1}{2}at^2$

(iii) $v^2 = u^2 + 2as$

**Group A**

Use the relations $\underline{a} = \frac{\underline{v} - \underline{u}}{t}$ and $s = \frac{(\underline{u} + \underline{v})}{2}t$

**Group B**

If the initial velocity is $\underline{u}$, acceleration is $\underline{a}$, final velocity is $\underline{v}$, time is $t$ and displacement is $s$m construct a velocity - time curve. Use the gradient and the area between the curve and the time axis.

**Group C**

When in vertical motion under constant gravitational acceleration $g$ use the relations $\underline{a} = \frac{\underline{v} - \underline{u}}{t}$ and $s = \frac{(\underline{u} + \underline{v})}{2}t$ .

**Group D**

When in vertical motion constant gravitational acceleration $g$, it is initial velocity is $\underline{u}$, acceleration is $\underline{g}$, final velocity is $\underline{v}$, time is $t$ and displacement is $s$, construct a velocity-time curve. Use gradient and the area between the curve and the time axis.

**Question**

| | |
|---|---|
| Group A and B | If is $\underline{a} = -2ms^{-2}$, $\underline{v} = 5ms^{-1}$, $t = 3s$ then find $\underline{u}$ and $\underline{s}$ |
| Group C and D | If is $\underline{s} = 490m$, $t$=10 s, $\underline{v} = 98ms^{-1}$ then find $\underline{u}$ and $\underline{a}$ |

# Grade 12 Third Term - Evaluation Plan - 1
# (Combined Maths - II)

**01. Competency** : 3. Perceives natural motions taken place on a plane using the Newtonians model for motion.

**Competency Level** : 3.5 Determines the motion of a particle on a plane relative to another particle.

**02. Nature of Evaluation Plan** : It is a group activity of inquiring into the relativity of motion.

**03. Time** : 90 minutes

**04. Instructions for the implementation of the evaluation plan :**

1. Three copies of the work sheet in annexure 1.
2. Demi papers, Maker pens should be supplied.

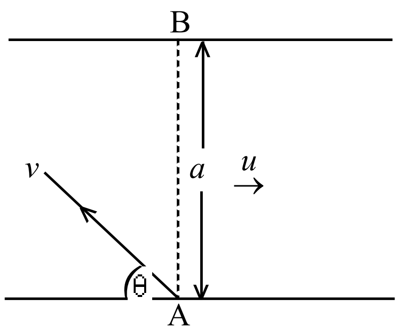
**Step 1 :**
- (i) Divide the class into three groups A, B and C.
- (ii) Provide each group with a copy of work sheet.
- (iii) Supply demi papers and marker pens to the groups.
- (iv) Engage the groups in the task.
- (v) Prepare the groups to present explanations to the class.

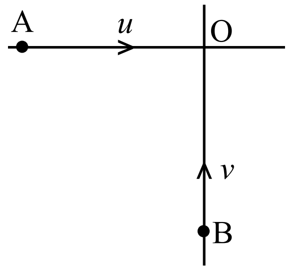
**05. Criteria for Evaluation :**

1. Expresses the principles of relative motion describing the motion of one particle relative to each particle on a plane.
2. Appreciates the applicability of the principles of relative motion in studying a motion.
3. Applies the principles of relative motion in solving problems on motion.
4. Applies theoritical models for solving practical problems.
5. Look at some events objectively from all angles.

**Work Sheet**

- Three events relating to relative motion are given below. You are required to select the event relevant to your group and study it under the following topics.
    - Event
    - Problem
    - Information leading to the solution of the problem.
    - Solution
    - Assumptions made
    - Practical importance
- Make use of the guidelines provided in studying the problem.

| Group | Event |
|---|---|
| A | <br>A river of width $a$ with parallel banks is following with a velocity $u$. A and B are two ferries on the opposite banks. Where AB = a. A ferryman rows his boat with a velocity $v$ intending to reach B from A. Assume that he is rowing the boat pointed at an angle $\theta$ with the upward direction of the river.<br>Discuss the situations<br>(i) $u > v\cos\theta$<br>(ii) $u = v\cos\theta$ |
| B | On a day when the wind is blowing steadily, for a cyclist travelling north with a velocity $v$, the wind appears to be flowing from east.<br>(i) Inquire whether the data given above is sufficient to determine the true velocity of the wind and if not state the additional data required.<br>(ii) Stating the factors affecting the apparent direction of the wind discuss whether those factors should act collectively or otherwise in order to change the apparent direction. |

| | |
|---|---|
| C |  Two perpendicular linear highways intersect at O. In these highways a motor car A is travelling with a velocity $u$ on one way and another motor car B is travelling with velocity $v$ on the other way towards O.<br><br>Let $AO = d_1$; $BO = d_2$ at the time $t$.<br>Discuss the conditions necessary to avoid a mishap.<br>(i) In terms of the principles of relative motion.<br>(ii) by any other method. |

- Be prepared to present your explanations to the class.

# Grade 12 Third Term - Evaluation Plan - 2
## (Combined Maths - II)

**01. Competency :** 3. Perceives natural motions taking place on a plane using the Newtonian model on motion.

**Competency Level :** 3.5 Applies vector methods to solve problems on relative motion.

**02. Nature of Evaluation Plan :** It is a group activity about simple impact of smooth elastic spheres.

**03. Time** : 90 minutes

**04. Instructions for the implementation of the evaluation plan :**

It is a group activity that can be implemented under the theme simple impacts of smooth elastic spheres. The final outcome will be a wall newspaper denoting the ratios of approaching and departing velocities and the momentums before and after impact.

(a) Prerequisites and instructions.

(i) On a day before implementing the activity discuss with the students practical situations where impact of elastic spheres occur and their aims. Give them a knowledge of conservation of momentum, approaching and departing relative velocities.

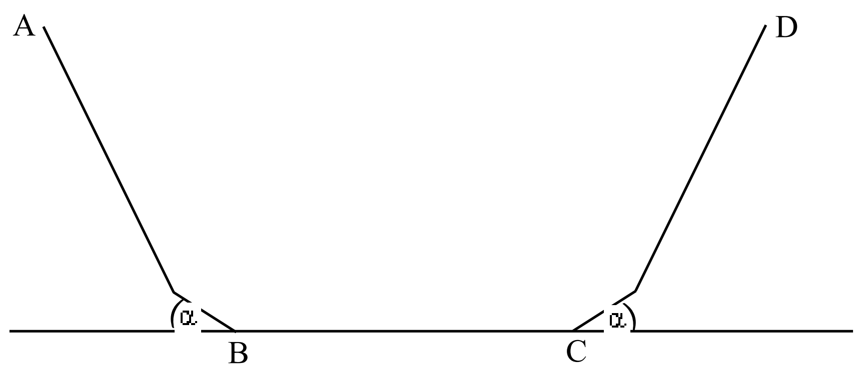
(ii) Group the students with about 5 in each group.

(iii) Prepare sufficient work sheets for the groups when there are several groups use parallel work sheets.

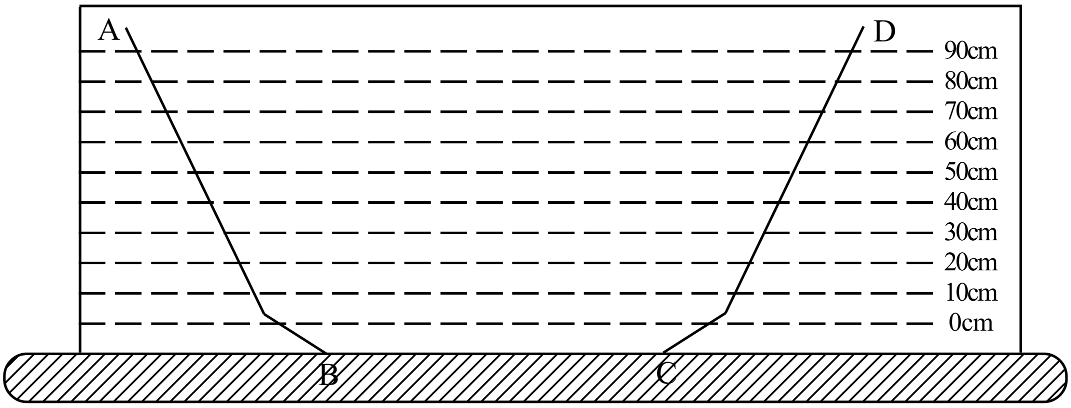
(iv) Create an awareness among the students about the material and information to be supplied for the task and the criteria for evaluation.

(v) Make them aware how a motion of a ball takes place in a groove and how the motion accures after the impact.

(vi) Take a grooved aluminium hood used for hanging curtains and bend it symmetrically. B and C should take the form of a curve.



Then fix it to a vertical board by means of clips supplied with it with BC horizontal and the plane of the rod vertical. Then graduate in *cm* the vertical height from the level BC on the board.




(vii) Obtain small plastic balls, glass balls, iron balls, ball bearings of various radii. These balls should be able to move freely in the groove H.

(viii) Prepare materials for each groups as follows.

**Group 1**

A board as above, three spheres of same radius and mass, a spring balance suitable for measuring the masses of the spheres.

**Group 2**

A board as above, two spheres of same radius and different masses, a spring balance suitable to measure the masses of spheres.

**Group 3**

A board as above, two spheres of different radii and masses, a spring balance to measure the masses of the spheres.

**Procedure**

(i) Supply each group with work sheets and engage them in the activity.

(ii) Watch their activity and give them the necessary instructions and guidelines.

(iii) At the end of the activity give them an opportunity to discuss among the groups their findings.

## Work Sheet for Group 1

- Let them place the two balls supplied on the arms AB and DC at the same or different heights and release them freely so as to cause them to collide in the part BC. Your group is required to observe the way of motion after the impact.

(1) Read and understand this work sheet and discuss about the task with the other members in the group.

(2) Obtain the materials supplied.

(3) Name the two spheres as P and Q and measure their masses.

(4) Place on arms AB and DC spheres P and Q at the same or different heights and release them freely so that they collide on BC. Measure the heights $h_1$ and $h_2$. Repeat by changing the heights $h_1$ and $h_2$.

(5) After the impact of P and Q on BC try to obtain the following situations of motion.

(i) After the impact both spheres move together in the same direction.

(ii) After impact they move separately in the same direction.

(iii) After impact they move separately in opposite directions.

(6) After the impacts (i), (ii) and (iii), obtain the heights $h_1$ and $h_2$ the spheres P and Q ascended along the arms BA and CD.

(7) Repeat this process several times.

(8) Taking $v_2 = \sqrt{2gh}$ calculate the velocity corresponding to the height $h$ and tabulate as shown below.
Calculate the velocities $v_1$, $v_2$, $v_3$ and $v_4$ corresponding to the heights $h_1$, $h_2$, $h_3$ and $h_4$ respectively.

Complete the following table.

**Table I**

| | $h_1$ | $h_2$ | $v_1$ | $v_2$ | $h_3$ | $h_4$ | $v_3$ | $v_4$ | | | $\frac{v}{u}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1<br>2<br>3<br>4<br>...<br>...<br>...<br>...<br>... | | | | | | | | | | | |

(9) State your conclusion as to the value of $\frac{v}{u}$.

(10) Since the masses of P and Q are known complete the Table II below.

**Table II**

| | Before impact | | After impact | | Momentum of system before impact | Momentum of system after impact |
|---|---|---|---|---|---|---|
| | Momentum of P | Momentum of Q | Momentum of P | Momentum of Q | | |
| 1<br>2<br>3<br>4<br>5<br>... | | | | | | |

What is your conclusion about the momentums before and after impact from the table alone?

**Work Sheet for Group 2**

Same work sheet as for group 1 is used. But the two spheres are of same radius and different masses.

**Work Sheet for Group 3**

Same work sheet. Spheres are of different radii and masses.

**05. Criteria for Evaluation :**

1. Readiness, process and traits relevant to the assignment are expected to be evaluated.
2. Marks for the criteria in the evaluation model may be assigned groupwise or personally. The group mark for a criterion will be the personal mark of each student for that criterion.
3. Assign mark for criteria as follows.
   Excellent - 4 Very good - 3
   Good - 2 Average - 1
   Maximum mark entitled for a student is $16 \times 4 = 64$

**Evaluation Model**

| Criteria | Student Names | | |
|---|---|---|---|
| | Group 1 | Group 2 | Group 3 |
| (01) Readiness<br>(i) Energetic participation in discussion.<br>(ii) Exchange of ideas with the teacher.<br>(iii) Exchange of ideas with other students.<br>(iv) Contribution towards planning of activity.<br><br>(02) Engaging in the process<br>(i) Measuring masses of spheres.<br>(ii) Choosing $h_1$ and $h_2$ suitably.<br>(iii) Measuring time.<br>(iv) Measuring $h_1$, $h_2$, $h_3$ and $h_4$.<br><br>(03) Finish<br>(i) Calculating $v_1$, $v_2$, $v_3$, $v_4$<br>(ii) Completing Table I<br>(iii) Completing Table II<br>(iv) Conclusion about $\frac{v}{u}$<br>(v) Conclusion on momentums before and after impact.<br><br>(04) Traits exhibited by the student from the begining to the end of the assignment.<br>(i) Following teacher instructions.<br>(ii) Interest shown in taking measurements.<br>(iii) Co-operation with the group.<br>(iv) Tolerating opinions of others. | | | |

## Grade 12 Third Term - Evaluation Plan - 3
## (Combined Maths - II)

**01. Competency** : 2. Interprets the system of coplanar forces in the meaningful application of it in the day to day life.

**Competency Level** : 2.9 Investigates the effect of friction on equilibrium in terms of its sign.

**02. Nature of Evaluation Plan** : Is a group activity of investigating the equilibrium of a body on a rough surface.

**03. Time** : 90 minutes

**04. Instructions for the implementation of the evaluation plan :**

1. Three copies of work sheet in annexure 1
2. Triple beam balance, protractor, pully, balance pan, strings and weights.
3. Some weights of low height.
4. Demi papers.
5. Marker pens, should be supplied.

**Step 1 :** (i) Divide the students into 3 groups A, B and C.
(ii) Provide each group with a copy of work sheet.
(iii) Get them engaged in the group activity.
(iv) Prepare them to present the explanations to the class.

**05. Criteria for Evaluation :**

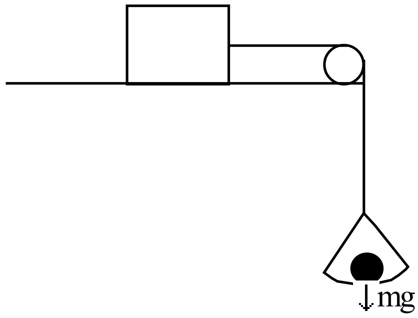
1. Expresses the conditions for the equilibrium of a body on a rough surface.
2. Realises that the frictional force is a quality endemic to the nature of the surfaces in contract.
3. Finds the conditions for the equilibrium of a body under frictional force.
4. Makes use of contradictions of an event suitably.
5. Applies suitable techniques to reach decisions.

*Annexure 1*

**Work Sheet for Group 1**

- Engage in the activity assigned to you.
- Be prepared to present group explanations.
- Discuss your observations within the group and reach reasonable conclusions.

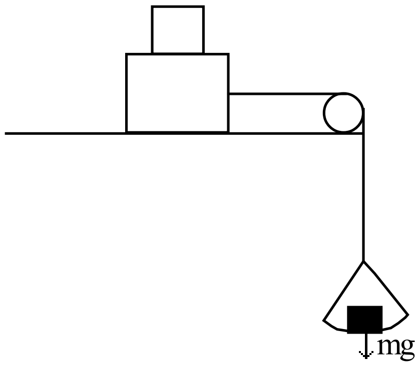
**Group A**




(i) Take several pieces of wood of different shapes and areas taken from the same board. Using an arrangement as shown above find the minimum force required to move each piece of wood on the table. Find $\frac{F\ lim}{R}$ for each case.

(ii) Repeat this for bodies made of surfaces with various materials and determine $\frac{F\ lim}{R}$ for each. Analyse the results obtained in the two occasions.

**Group B**




Using an arrangement as shown above find the minimum force necessary to just move a piece of wood placed on a horizontal table. Repeating this for pieces of wood of increasing weights placed on it and determine the minimum force necessary to just move each. Draw a graph between the weight used to move and the total weight on the board with the board and analyse the results.

**Group C**

Place a small piece of wood on a plane and increase its inclination to the horizontal gradually. When the piece of wood is just about to move measure the inclination of the plane to the horizontal. Repeat this by placing decreasing weights on the piece of wood. Further measure the angles by replacing the piece of wood with bodies made of different materials. Analyse the results obtained.